MATH 141
FALL 2019
EXAM 3

NAME:

1. $\boxed{\text{10 pts. each}}$ Find the limit of each sequence, or show that the limit does not exist.

   (a) $\left(\ln \sin \frac{1}{n} + \ln n\right)$

   (b) $\left(\sqrt[n]{e^{3n+4}}\right)$

2. $\boxed{\text{10 pts. each}}$ Suppose the sequence $(a_n)_{n=0}^{\infty}$ is defined by the recurrence relation

$$a_{n+1} = \frac{1}{3}a_n + 6, \quad a_0 = 3.$$

   (a) Prove that the sequence is increasing and bounded.

   (b) Explain why (a_n) converges and find the limit.

3. $\boxed{\text{10 pts.}}$ Evaluate the geometric series or explain why it diverges:

$$\sum_{n=1}^{\infty} \frac{8}{4^n}.$$

4. $\boxed{\text{10 pts.}}$ Either show the telescoping series

$$\sum_{n=1}^{\infty} \frac{20}{25n^2 + 15n - 4}$$

diverges, or evaluate the series.

5. $\boxed{\text{10 pts.}}$ Write the repeating decimal $5.1\overline{32}$ as a geometric series and then as a fraction (i.e. a ratio of integers).

6. $\boxed{\text{10 pts. each}}$ Determine whether the series converges or diverges, using an appropriate test and justifying all work. Arguments must be clear and thorough.

   (a) $\sum_{n=1}^{\infty} \frac{4^n}{5^n - 6}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{2^{\ln n}}$

   (c) $\sum_{k=1}^{\infty} \left(\frac{k^2}{2k^2 + 1}\right)^k$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)! - n!}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}e^{\sqrt{n}}}$

(f) $1 + \dfrac{1 \cdot 3}{3!} + \dfrac{1 \cdot 3 \cdot 5}{5!} + \dfrac{1 \cdot 3 \cdot 5 \cdot 7}{7!} + \cdots$

7. $\boxed{\text{10 pts. each}}$ Use the Alternating Series Test to show the series converges, or use some other test to show it diverges. If the series converges, use any test to determine whether it converges absolutely or conditionally.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2\sqrt{k} - 1}$

Some Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$