MATH 141
FALL 2019
EXAM 2

NAME:

1. [10 pts. each] Evaluate the integral by integrating by parts.

(a) $\int x^2 \cos 4x \, dx$

(b) $\int e^{-x} \sin 3x \, dx$

2. [10 pts.] Find the arc length of the curve given by the function

$$f(x) = \int_e^x \sqrt{\ln^2 t - 1} \, dt$$

on $[e, e^4]$.

3. [10 pts. each] Evaluate the trigonometric integral.

(a) $\int \sin^5 6t \cos^2 6t \, dt$

(b) $\int_{\pi/16}^{\pi/8} \csc^2 4r \cot^4 4r \, dr$

4. [10 pts. each] Evaluate using trigonometric substitution.

(a) $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4 - x^2}} \, dx$

(b) $\int \frac{dt}{t^2 \sqrt{t^2 - 16}}$

5. [10 pts. each] Evaluate using partial fractions.

(a) $\int_1^2 \frac{7x - 2}{3x^2 - 2x} \, dx$

(b) $\int \frac{x^4 + 1}{x^3 + 9x} \, dx$

6. [10 pts. each] Evaluate using any strategy.

(a) $\int \frac{1}{e^x \sqrt{1 + e^{2x}}} \, dx$

(b) $\int s^2 \tan^{-1} s \, ds$

7. 10 pts. each Determine whether the integral is convergent or divergent, and evaluate if convergent.

(a) $\displaystyle\int_{-\infty}^{-1} \frac{dx}{\sqrt[3]{x}}$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{(\tan^{-1} t)^2}{t^2+1}\, dt$

(c) $\displaystyle\int_{-3}^{3} \frac{dq}{\sqrt{9-q^2}}$

8. 10 pts. Use the Comparison Theorem to determine whether the integral converges or diverges:

$$\int_0^{\infty} \frac{dx}{e^x + x + 1}.$$

Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx$
- $\displaystyle\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx$
- $\displaystyle\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx$
- $\displaystyle\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$
- $\displaystyle\int \tan x\, dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\, dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\, dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\, dx = -\ln|\csc x + \cot x| + c$