MATH 141
FALL 2019 NAME:
EXAM 1

1. [10 pts.] Find all inverses associated with the function

$$f(x) = \frac{2}{x^2 + 2},$$

and state their domains.

2. [10 pts.] Find $(f^{-1})'(3)$ for the function $f(x) = x^3 + x + 1$.

3. [10 pts. each] Differentiate the function.

(a) $f(x) = \ln\left(\dfrac{x+1}{x-1}\right)$

(b) $F(x) = \dfrac{e^{6x}}{e^{-x} + 12}$

(c) $s(t) = 8^{\sec 2t}$

(d) $g(x) = x^{\ln x}$

(e) $u(x) = \log_{10}(10x - 1)$

(f) $h(r) = (\ln r)^{\cos r}$

(g) $q(z) = \tan^{-1}(e^{10z})$

(h) $p(x) = -\sinh^3 4x$

4. [15 pts.] Find an equation of the tangent line to $y = \ln(x^3 - 7)$ at the point $(2, 0)$.

5. [10 pts. each] Evaluate each integral.

(a) $\displaystyle\int_2^4 \frac{2}{16 - 3t}\,dt$

(b) $\displaystyle\int \frac{dx}{x \ln x \ln(\ln x)}$

(c) $\displaystyle\int \frac{\log_2 z^{10}}{z}\,dz$

(d) $\displaystyle\int_1^2 (1 + \ln x)x^x\,dx$

(e) $\displaystyle\int \frac{e^x}{e^{2x} + 4}\,dx$

(f) $\displaystyle\int_0^1 \cosh^3 3y \sinh 3y\,dy$

6. [10 pts.] Find the length of the catenary $y = \cosh x$ over the interval $[0, a]$.

7. [10 pts. each] Evaluate the limit using L'Hôpital's Rule.

   (a) $\lim_{x\to 0} \dfrac{\sin^{-1} x}{x}$

   (b) $\lim_{x\to 0} (x + \cos x)^{1/x}$

   (c) $\lim_{x\to 0^+} (a^x - b^x)^x$, $a > b > 0$

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$