MATH 141
FALL 2018 NAME:
EXAM 2

1. [10 pts.] Find the area of the region bounded by the curves

$$y = \frac{x^2}{x^3 - 3x} \quad \text{and} \quad y = \frac{1}{x^3 - 3x}$$

on the interval $[2, 4]$.

2. [10 pts. each] Evaluate each of the following.

(a) $\displaystyle\int \theta \sec^2 \theta \, d\theta$

(b) $\displaystyle\int x^2 \sin 2x \, dx$

3. [10 pts.] Use a substitution followed by integration by parts to evaluate

$$\int (\sec^2 x) \ln(2 + \tan x) \, dx$$

4. [10 pts. each] Evaluate each integral.

(a) $\displaystyle\int \sin^2 x \cos^2 x \, dx$

(b) $\displaystyle\int_0^{\pi/4} (1 + \cos 4x)^{3/2} \, dx$

5. [10 pts. each] Use a trigonometric substitution to find the indefinite integral

(a) $\displaystyle\int \sqrt{9 - 4x^2} \, dx$

(b) $\displaystyle\int \frac{r^4}{1 + r^2} \, dr$

6. [10 pts. each] Use partial fractions to find the indefinite integral

(a) $\displaystyle\int \frac{x^2 + 12x - 4}{x^3 - 4x} \, dx$

(b) $\displaystyle\int_1^2 \frac{2}{t^3(t + 1)} \, dt$

7. [10 pts.] Evaluate the improper integral or state that it diverges:

$$\int_0^\infty e^{-ax} \, dx, \quad a > 0.$$

8. [10 pts.] Evaluate the improper integral, or state that it diverges:

$$\int_0^1 \frac{t^3}{t^4 - 1}\, dt.$$

9. [10 pts.] Let $\mathcal{R}$ be the region on the xy-plane bounded by

$$f(x) = \frac{1}{\sqrt{x}\ln x}$$

and the x-axis on the interval $[2, \infty)$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1 - x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1 + x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2 - 1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx$
- $\displaystyle\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx$
- $\displaystyle\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx$
- $\displaystyle\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$
- $\displaystyle\int \tan x\, dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\, dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\, dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\, dx = -\ln|\csc x + \cot x| + c$