

MATH 141
FALL 2018
EXAM 1

NAME:

1. Consider the function $f(x) = 6x + \sin x$.
 - (a) [5 pts.] Show that f is one-to-one and hence has an inverse.
 - (b) [10 pts.] Find $(f^{-1})'(0)$ using the Inverse Function Theorem
2. [10 pts. each] Find the derivative of each function.
 - (a) $y = \frac{x}{\ln x - 2}$
 - (b) $f(x) = \tan(\cos(e^x))$
 - (c) $g(x) = (\cot x)^{x-1}$
 - (d) $h(t) = t^{(t^{1/2})}$
 - (e) $\ell(x) = 4 \log_3(1 - x^3)$
 - (f) $\varphi(u) = \tan^{-1}(4u - 4)$
 - (g) $y = x^2 \cosh^3 5x$
3. [10 pts.] Find an equation of the line tangent to $y = 2e^x + 8$ at the point where $x = \ln 6$.
4. [10 pts. each] Determine each indefinite integral.
 - (a) $\int x^5 e^{x^6} dx$
 - (b) $\int \frac{1}{(x \ln x) \ln(\ln x)} dx$
 - (c) $\int \frac{e^{\cos x}}{\csc x} dx$
5. [10 pts. each] Evaluate each definite integral.
 - (a) $\int_{-2}^2 \frac{e^{t/2}}{e^{t/2} + 1} dt$
 - (b) $\int_0^{3/2} \frac{1}{\sqrt{36 - 4x^2}} dx$
 - (c) $\int_0^1 \cosh^3 4x \sinh 4x dx$
6. [10 pts.] Evaluate the limit using L'Hôpital's Rule: $\lim_{x \rightarrow \infty} (2x^3 + 1)^{1/\ln x}$.

FORMULAS & DEFINITIONS

- $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \frac{1}{1+x^2}$
- $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$
- $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$
- $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
- $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$
- $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$
- $\int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$
- $\int \tan x dx = \ln |\sec x| + c$
- $\int \cot x dx = \ln |\sin x| + c$
- $\int \sec x dx = \ln |\sec x + \tan x| + c$
- $\int \csc x dx = -\ln |\csc x + \cot x| + c$