MATH 141
FALL 2017 NAME:
EXAM 2

1. [10 pts.] Use integration by parts to determine

$$\int t^3 \ln t \, dt$$

2. [10 pts.] Use integration by parts to determine

$$\int x \sec^2 x \, dx$$

3. [10 pts.] Use integration by parts to evaluate

$$\int_{2/\sqrt{3}}^{2} x \sec^{-1} x \, dx$$

4. [10 pts.] Evaluate

$$\int_0^{\pi/2} \sin^{5/2} x \cos^3 x \, dx$$

5. [10 pts.] Evaluate

$$\int_0^{\pi/4} \sec^4 \theta \, d\theta$$

6. [10 pts.] Use a trigonometric substitution to evaluate

$$\int_0^{1/3} \frac{3}{\sqrt{1+9y^2}} \, dy.$$

7. [10 pts.] Use a trigonometric substitution to determine

$$\int \sqrt{25 - t^2} \, dt.$$

8. [10 pts. each] Use partial fractions to find the indefinite integral.

(a) $\displaystyle\int \frac{12r}{(r-4)^2} \, dr$

(b) $\displaystyle\int \frac{x+1}{x^2(x-2)} \, dx$

9. [10 pts. each] Evaluate the improper integral, or show that it diverges.

   (a) $\displaystyle\int_2^\infty \frac{1}{y \ln y}\, dy$

   (b) $\displaystyle\int_0^3 \frac{1}{(x-1)^{2/3}}\, dx$

10. [10 pts.] Let R be the region between the x-axis and the curve $y = e^{-x}$ for $x \geq 0$. Find the volume of the solid generated by revolving R about the x-axis.

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx$
- $\displaystyle\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx$
- $\displaystyle\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx$
- $\displaystyle\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$
- $\displaystyle\int \tan x\, dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\, dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\, dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\, dx = -\ln|\csc x + \cot x| + c$