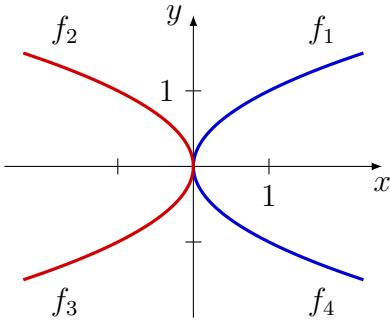


MATH 141
 FALL 2017
 EXAM 1

NAME:

1. [10 pts. each] The equation $y^4 = 4x^2$ gives rise to four one-to-one functions f_1, f_2, f_3, f_4 , where the graph of f_k is in the k th quadrant for each $1 \leq k \leq 4$ (see figure below).

- (a) Find the domain and a formula for each function f_k .
 (b) Find the inverse of each function and write it as $y = f_k^{-1}(x)$.



2. [15 pts.] Consider the function $f(x) = x - \pi + \cos x$, $0 < x < 2\pi$. Show that f is one-to-one and hence has an inverse, then find $(f^{-1})'(-1)$ using the Inverse Function Theorem.
3. [10 pts. each] Find the derivative of each function.
- (a) $y = \frac{\ln x}{\ln x + 1}$
 (b) $f(x) = \sin(\cos(e^x))$
 (c) $g(x) = (\tan x)^{x-1}$
 (d) $h(t) = t^{(t^{10})}$
 (e) $\ell(x) = 4 \log_3(1 - x^3)$
 (f) $\varphi(u) = \csc^{-1}(4u - 4)$
 (g) $y = x^2 \cosh^3 5x$

4. [10 pts.] Find an equation of the line tangent to $y = 2e^x - 1$ at the point where $x = \ln 3$.

5. [10 pts. each] Determine each indefinite integral.

- (a) $\int x^5 e^{x^6} dx$
 (b) $\int \frac{1}{(x \ln x) \ln(\ln x)} dx$
 (c) $\int \frac{e^{\sin x}}{\sec x} dx$

6. [10 pts. each] Evaluate each definite integral.

$$(a) \int_{-2}^2 \frac{e^{t/2}}{e^{t/2} + 1} dt$$

$$(b) \int_0^{3/2} \frac{1}{\sqrt{36 - 4x^2}} dx$$

$$(c) \int_0^1 \cosh^3 4x \sinh 4x dx$$

7. [10 pts.] Evaluate the limit using L'Hôpital's Rule: $\lim_{x \rightarrow \infty} (x^3 + 1)^{1/\ln x}$

FORMULAS & DEFINITIONS

- $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \frac{1}{1+x^2}$
- $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$
- $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$
- $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
- $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$
- $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$
- $\int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$
- $\int \tan x dx = \ln |\sec x| + c$
- $\int \cot x dx = \ln |\sin x| + c$
- $\int \sec x dx = \ln |\sec x + \tan x| + c$
- $\int \csc x dx = -\ln |\csc x + \cot x| + c$