MATH 141
FALL 2016
EXAM 4

NAME:

1. $\boxed{\text{10 pts.}}$ Approximate the quantity $\sqrt[5]{31}$ using a 3rd-order Taylor polynomial centered at 32.

2. $\boxed{\text{10 pts. each}}$ Determine the interval of convergence of the power series, making sure to test endpoints.

   (a) $\sum \frac{n^3 x^{4n}}{n!}$ (b) $\sum \frac{(-1)^{n-1} x^n}{n^3}$ (c) $\sum \frac{(-2)^n}{\sqrt[4]{n}}(x-1)^n$

3. $\boxed{\text{10 pts.}}$ Find the function represented by the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n x^{n+1}}{6^n}.$$

4. $\boxed{\text{5 pts. each}}$ Let $f(x) = \ln(x)$.

   (a) Find the first four nonzero terms of the Taylor series for f centered at 1.

   (b) Write the Taylor series using summation notation.

   (c) Find the radius of convergence and interval of convergence.

5. $\boxed{\text{10 pts.}}$ Use a Taylor series to approximate the value of the definite integral

$$\int_0^{0.5} \frac{1}{\sqrt{1+x^6}}\,dx$$

   with an absolute error less than 10^{-3}.

6. $\boxed{\text{10 pts.}}$ Consider the parametric equations

$$x = \frac{3}{t+5} - 2, \quad y = t+1; \quad 0 \le t \le 10.$$

   Eliminate the parameter to obtain an equation of the form $y = f(x)$. What is the domain of f?

7. $\boxed{\text{10 pts.}}$ Find a parametric description of the line containing the points $(-1, 0)$ and $(0, 5)$.

8. $\boxed{\text{10 pts.}}$ Convert the polar equation $r = 2\sin\theta + 2\cos\theta$ to Cartesian coordinates.

9. $\boxed{\text{10 pts.}}$ Find the slope of the tangent line to the polar curve $r = 4\cos\theta$ at the point $(2, \frac{\pi}{3})$.

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \leq b_{k+1} \leq b_k$ for all k, then $R_n \leq b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \leq 1$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n, \text{ for } |x| < 1, \text{ where } \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!} \text{ and } \binom{p}{0} = 1.$$

Some Trigonometric Identities:

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$