1. ⟦10 pts.⟧ Find the integral:

$$\int \frac{1}{x^{1/2} + x^{3/2}} \, dx.$$

2. ⟦10 pts.⟧ Use integration by parts to evaluate

$$\int_0^{\ln 2} x e^{-4x} \, dx.$$

3. ⟦10 pts.⟧ Determine

$$\int \sin \sqrt{x} \, dx$$

using a substitution followed by integration by parts.
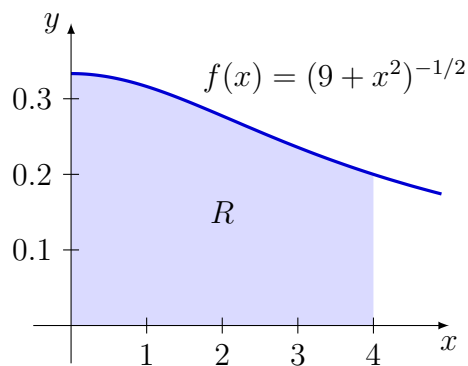
4. ⟦10 pts.⟧ Determine the trigonometric integral

$$\int \tan^3 x \sec^2 x \, dx.$$

5. ⟦10 pts.⟧ Use a trigonometric substitution to evaluate

$$\int_6^{6\sqrt{3}} \frac{q^2}{(q^2 + 36)^2} \, dq.$$

6. ⟦10 pts. each⟧

(a) Find the area of the region R bounded by the curves $f(x) = (9 + x^2)^{-1/2}$, $x = 0$, $x = 4$, and $y = 0$. See figure below.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

7. [10 pts. each] Use partial fractions to find the indefinite integral.

(a) $\displaystyle\int \frac{12}{(r-4)(r+3)}\, dr$

(b) $\displaystyle\int \frac{x-5}{x^2(x+1)}\, dx$

8. [10 pts. each] Evaluate the improper integral, or show that it diverges.

(a) $\displaystyle\int_{-\infty}^{-2} \frac{2}{t^2-1}\, dt$

(b) $\displaystyle\int_{0}^{1} \frac{x^3}{x^4-1}\, dx$

Formulas

- $\tan x = y \Leftrightarrow \tan^{-1} y = x, \quad$ for $x \in (-\pi/2, \pi/2)$
- $\cot x = y \Leftrightarrow \cot^{-1} y = x, \quad$ for $x \in (0, \pi)$
- $\sec x = y \Leftrightarrow \sec^{-1} y = x, \quad$ for $x \in [0, \pi/2) \cup (\pi/2, \pi]$
- $\csc x = y \Leftrightarrow \csc^{-1} y = x, \quad$ for $x \in [-\pi/2, 0) \cup (0, \pi/2]$
- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \dfrac{1}{1+x^2}, \quad (\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int a^x\, dx = \frac{a^x}{\ln a} + c$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx$
- $\displaystyle\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx$
- $\displaystyle\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx$
- $\displaystyle\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$
- $\displaystyle\int \tan x\, dx = \ln|\sec x| + c, \quad \int \cot x\, dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\, dx = \ln|\sec x + \tan x| + c, \quad \int \csc x\, dx = -\ln|\csc x + \cot x| + c$