Math 141
Fall 2015
Final Exam

Name:

1. Find the derivative of each function.

   (a) $f(x) = \ln(x + \ln x)$

   (b) $g(t) = te^{3t^2}$

   (c) $y = \sin^{-1}(x^4)$

2. Evaluate each integral.

   (a) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

(b) $\displaystyle\int \frac{4}{x^2+3}\,dx$

(c) $\displaystyle\int_1^{e^2} x^2 \ln x\,dx$

(d) $\displaystyle\int \sin^5 x \cos^{-2} x\,dx$

(e) $\displaystyle\int \frac{1}{(1+4x^2)^{3/2}}\,dx$

(f) $\displaystyle\int \frac{2}{x^2-x-6}\,dx$

3. Evaluate the integral, or state that it diverges: $\displaystyle\int_0^\infty e^{-5x}\,dx$

4. Find the limit of the sequence, or state that it diverges: $\displaystyle\left\{\left(1+\frac{2}{n}\right)^n\right\}$

5. Evaluate the series, or state that it diverges: $\sum_{k=1}^{\infty} \frac{5}{3^k}$

6. Determine whether the series converges or diverges, giving a clear justification for your answer using any one of the following: Divergence Test, Integral Test, Comparison Test, Limit Comparison Test, Ratio Test, Root Test, Alternating Series Test, or properties of the p-series. Specify the test being used.

   (a) $\sum_{k=0}^{\infty} \frac{k}{100k+3}$

   (b) $\sum_{k=1}^{\infty} \frac{k}{\sqrt{k^2+4}}$

(c) $\sum_{k=1}^{\infty} \frac{k^6}{k!}$

(d) $\sum_{k=1}^{\infty} \frac{k^2}{2^k}$

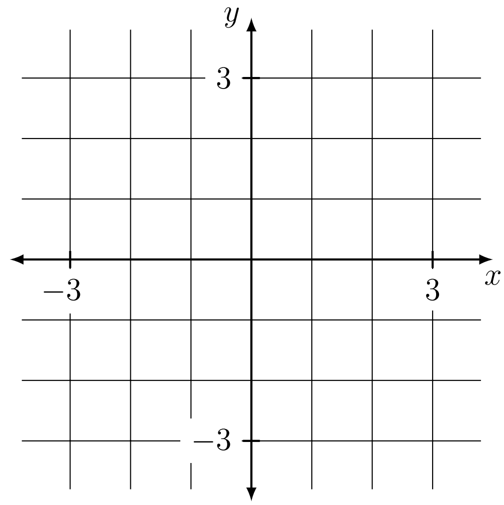
(e) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k^2+4}}$

7. Find the interval of convergence of the series $\sum_{n=0}^{\infty} \left(\frac{x}{8}\right)^{3n}$

8. Find the first 4 terms of the Maclaurin series expansion for $e^{x/3}$.

9. Find parametric equations for the circle centered at the origin with radius 6, generated counter-clockwise.

10. Plot the polar points $(2, \pi)$, $\left(3, -\frac{\pi}{4}\right)$, and $\left(-2, \frac{\pi}{3}\right)$ on the rectangular coordinate system below.



11. Convert the rectangular coordinates $\left(2, 2\sqrt{3}\right)$ to polar coordinates.

12. Convert the equation $r = \cot\theta \csc\theta$ to rectangular coordinates.