

1. 10 pts. Find the integral

$$\int \frac{1}{\sqrt{27 - 6\theta - \theta^2}} d\theta$$

(hint: complete the square).

2. 10 pts. each Use integration by parts to determine each of the following.

(a)  $\int e^x \sin x dx$

- (b) Let  $\mathcal{R}$  be the region bounded by  $f(x) = x \ln x$  and the  $x$ -axis on  $[1, e^2]$ . Find the volume of the solid generated by revolving  $\mathcal{R}$  about the  $x$ -axis.

3. 10 pts. each Find each indefinite trigonometric integral.

(a)  $\int (\cos^3 x) \sqrt{\sin x} dx$

(b)  $\int \frac{\sec^2 z}{\tan^5 z} dz$

(c)  $\int e^x \sec(e^x + 1) dx$

4. 10 pts. each Use a trigonometric substitution to find the indefinite integral

(a)  $\int_{1/2}^1 \frac{\sqrt{1-x^2}}{x^2} dx$

(b)  $\int \sqrt{169 - t^2} dt$

5. 10 pts. each Use partial fractions to find the indefinite integral

(a)  $\int \frac{8}{(y-4)^2(y+3)} dy$

(b)  $\int \frac{2}{(x-4)(x^2+2x+6)} dx$

6. 10 pts. Let  $\mathcal{R}$  be the region on the  $xy$ -plane bounded by  $f(x) = (x+1)^{-3}$  and the  $x$ -axis on the interval  $[0, \infty)$ . Find the volume of the solid generated by revolving  $\mathcal{R}$  about the  $y$ -axis, or show that it does not exist.

7. 10 pts. Evaluate the improper integral, or show that it diverges:

$$\int_{-1}^1 \ln(y^2) dy.$$

## FORMULAS & DEFINITIONS

1.  $\theta = \tan^{-1} x \Leftrightarrow x = \tan \theta$ , for  $\theta \in (-\pi/2, \pi/2)$
2.  $\theta = \cot^{-1} x \Leftrightarrow x = \cot \theta$ , for  $\theta \in (0, \pi)$
3.  $\theta = \sec^{-1} x \Leftrightarrow x = \sec \theta$ , for  $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4.  $\theta = \csc^{-1} x \Leftrightarrow x = \csc \theta$ , for  $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5.  $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$ , for  $x \in (-1, 1)$
6.  $(\tan^{-1} x)' = \frac{1}{1+x^2}$ , for  $x \in (-\infty, \infty)$
7.  $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$ , for  $x \in (-\infty, -1) \cup (1, \infty)$
8.  $\int a^x dx = \frac{a^x}{\ln a} + c$ , for  $a \in (0, 1) \cup (1, \infty)$
9.  $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$ , for  $a \in (0, \infty)$
10.  $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$ , for  $a \neq 0$
11.  $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$ , for  $a \in (0, \infty)$
12.  $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
13.  $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$
14.  $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$ ,  $n \neq 1$
15.  $\int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$ ,  $n \neq 1$
16.  $\int \tan x dx = -\ln |\cos x| + c = \ln |\sec x| + c$
17.  $\int \cot x dx = \ln |\sin x| + c$
18.  $\int \sec x dx = \ln |\sec x + \tan x| + c$
19.  $\int \csc x dx = -\ln |\csc x + \cot x| + c$