MATH 141
FALL 2015 NAME:
EXAM 1

1. [10 pts.] For the function $f(x) = -x^2 + 8$, find the slope of the line tangent to the graph of f^{-1} at the point $(7, 1)$.

2. [10 pts. each] Find the derivative of each function.
   (a) $f(x) = \cot(e^x)$
   (b) $g(x) = \ln(\ln x)$
   (c) $h(x) = (2x+1)^{2x}$
   (d) $x(t) = \sin(\sec^{-1} 2t)$
   (e) $v(y) = \cos^{-1}\left(\dfrac{1}{y^2+1}\right)$
   (f) $\varphi(x) = x^2 \cosh^2(3x)$

3. [10 pts. each] Determine each indefinite integral.
   (a) $\displaystyle\int (2e^{-6x} + e^{9x})\, dx$
   (b) $\displaystyle\int \frac{9}{4-9y}\, dy$
   (c) $\displaystyle\int x^7 8^{x^8}\, dx$
   (d) $\displaystyle\int \frac{\sinh t}{1+\cosh t} dt.$

4. [10 pts. each] Evaluate each definite integral.
   (a) $\displaystyle\int_1^2 (1+\ln x) x^x\, dx$
   (b) $\displaystyle\int_{-\ln\sqrt{3}}^0 \frac{e^x}{1+e^{2x}}\, dx$

5. [10 pts.] Evaluate the limit using L'Hôpital's Rule:

$$\lim_{x\to 1^+} \left(\sqrt{x-1}\right)^{2\sin \pi x}$$

Formulas & Definitions

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan\theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot\theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec\theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc\theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
6. $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
7. $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, 1) \cup (1, \infty)$
8. $\displaystyle\int b^x\,dx = \frac{1}{\ln b} b^x + c$, for $b \in (0, 1) \cup (1, \infty)$
9. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$, for $a \in (0, \infty)$
10. $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$, for $a \neq 0$
11. $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$, for $a \in (0, \infty)$
12. $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
13. $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
14. $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx,\ \ n \neq 1$
15. $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx,\ \ n \neq 1$
16. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + c = \ln|\sec x| + c$
17. $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
18. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
19. $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$