MATH 141
FALL 2011
EXAM 4

NAME:

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, for $|x| < \infty$

$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$, for $|x| < \infty$

$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$, for $|x| < \infty$

$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}$, for $-1 < x \leq 1$

$\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$, for $|x| \leq 1$

1. [15 pts. each] Determine the interval of convergence and radius of convergence of the power series, making sure to test endpoints.

   (a) $\sum_{k=0}^{\infty} \left(\frac{x+1}{8}\right)^k$

   (b) $\sum_{k=1}^{\infty} \frac{(2x+3)^k}{6k}$

2. [10 pts.] Use the geometric series

$$f(x) = \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad |x| < 1$$

   to find the power series representation (centered at 0) of the function $g(x) = \frac{5}{1-6x}$. Give the interval of convergence of the new series.

3. [10 pts.] Find the function represented by the series $\sum_{k=0}^{\infty} (\sqrt{x} - 7)^k$, and give the interval of convergence of the series.

4. Let $f(x) = \cos(4x)$.

   (a) [10 pts.] Find the first four nonzero terms of the Maclaurin series for f.

   (b) [5 pts.] Write the power series using summation notation.

   (c) [10 pts.] Determine the interval of convergence for the series.

5. [10 pts.] Evaluate

$$\lim_{x\to 0} \frac{3\tan^{-1} x - 3x + x^3}{x^5}$$

   using Taylor series. (Do *not* use L'Hôpital's Rule.)

6. [10 pts.] Use a Taylor series to approximate

$$\int_0^{0.15} \frac{\sin x}{x} dx,$$

   retaining as many terms as needed to ensure the error is less than 10^{-4}.

7. [10 pts.] Consider the parametric equations

$$x = \sqrt{t} + 4, \quad y = 3\sqrt{t}; \quad 0 \leq t \leq 16.$$

   Eliminate the parameter to obtain an equation in x and y.

8. [10 pts.] Express the Cartesian coordinates $(4, 4)$ in polar coordinates in two different ways.

9. [10 pts.] Convert the equation $r = 8\sin\theta$ to Cartesian coordinates, and describe the resulting curve.

10. [10 pts.] Find all points where the polar curve $r = \sin 2\theta$ has a horizontal tangent line.