MATH 141
FALL 2011
EXAM 3

NAME:

Remainder Theorem: Let $R_n = |S - S_n|$ be the remainder in approximating the value of a convergent alternating series $\sum_{k=1}^{\infty}(-1)^{k+1}a_k$ by the sum of its first n terms. Then $R_n \le a_{n+1}$.

1. $\boxed{\text{10 pts. each}}$ Find the limit of each sequence, or state that the limit does not exist.
   (a) $\left\{\dfrac{2n^{12}}{11n^{12}+4n^5}\right\}$
   (b) $a_n = (-1)^n \sqrt[n]{n}$
   (c) $\left\{\dfrac{\cos n}{2^n}\right\}$ (Use Squeeze Theorem)

2. $\boxed{\text{10 pts.}}$ Evaluate the geometric series $\displaystyle\sum_{k=2}^{\infty}\frac{5}{3^k}$.

3. $\boxed{\text{10 pts.}}$ For the telescoping series
$$\sum_{k=1}^{\infty}\frac{1}{(k+1)(k+2)},$$
find a formula for the nth term of the sequence of partial sums $\{s_n\}$, then evaluate $\lim_{n\to\infty} s_n$ to obtain the value of the series.

4. $\boxed{\text{10 pts.}}$ Use the Remainder Theorem given above to estimate the value of the convergent series
$$\sum_{k=1}^{\infty}\frac{(-1)^k}{k^k}$$
with an absolute error less than 10^{-3}.

5. $\boxed{\text{10 pts. each}}$ Determine whether the series converges or diverges. The test(s) you are allowed to use are indicated in parentheses.
   (a) $\displaystyle\sum_{k=0}^{\infty}\frac{k}{99k+50}$, (Divergence Test or either Comparison Test)
   (b) $\displaystyle\sum_{k=1}^{\infty}\frac{k}{\sqrt{k^2+4}}$, (any test that works)
   (c) $\displaystyle\sum_{k=1}^{\infty}\frac{(k!)^2}{(2k)!}$, (Ratio Test)
   (d) $\displaystyle\sum_{k=1}^{\infty}\frac{k^2}{2^k}$, (Root Test)
   (e) $\displaystyle\sum_{k=1}^{\infty}\frac{k^2-1}{k^3+9}$, (either Comparison Test)
   (f) $\displaystyle\sum_{k=1}^{\infty}\frac{k^8}{k^{11}+3}$, (any test that works)

6. $\boxed{\text{10 pts. each}}$ If a series converges, use the Alternating Series Test to show it; otherwise, use some other test to show divergence.
   (a) $\displaystyle\sum_{k=2}^{\infty}\frac{(-1)^k}{k\ln^2 k}$
   (b) $\displaystyle\sum_{k=2}^{\infty}(-1)^k\left(1+\frac{2}{k}\right)$