Math 141
Fall 2011
Exam 2

Name:

1. [10 pts.] Integrate, using a reduction formula if necessary: $\int \tan^3(4x)\,dx$

2. [10 pts.] Integrate, using a reduction formula if necessary: $\int \sin^5 x \cos^{-2} x\,dx$

3. [10 pts.] Integrate using a trigonometric substitution: $\int \frac{1}{\sqrt{x^2-81}}\,dx,\ x>9$

4. [10 pts.] Integrate using a trigonometric substitution: $\int \frac{1}{(1+4x^2)^{3/2}}\,dx$

5. [10 pts.] Integrate using partial fractions: $\int \frac{2}{x^2-x-6}\,dx$

6. [10 pts.] Integrate using partial fractions: $\int \frac{y}{(y-6)(y+2)^2}\,dy$

7. [10 pts.] Integrate using partial fractions: $\int \frac{z+1}{z(z^2+4)}\,dz$

8. [10 pts.] Evaluate the improper integral, or show that it diverges: $\int_0^\infty e^{-5x}\,dx$

9. [10 pts.] Evaluate the improper integral, or show that it diverges: $\int_0^1 \frac{x^3}{x^4-1}\,dx$

10. [5 pts. each] For the sequence $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots\right\}$ do the following.
    (a) Find the next two terms of the sequence.
    (b) Find a recurrence relation that generates the sequence.
    (c) Find an explicit formula for the general nth term of the sequence.

Formulas & Definitions

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan\theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot\theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec\theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc\theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $\dfrac{d}{dx}(b^x) = b^x \ln b$, for $b \in (0,1) \cup (1, \infty)$
6. $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
7. $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
8. $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, 1) \cup (1, \infty)$
9. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$
10. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$, for $a \in (0, \infty)$
11. $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$, for $a \neq 0$
12. $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$, for $a \in (0, \infty)$
13. $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
14. $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
15. $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx,\ \ n \neq 1$
16. $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx,\ \ n \neq 1$
17. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$
18. $\displaystyle\int \cot x\,dx = \ln|\sin x| + C$
19. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$
20. $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + C$