Math 141
Fall 2011
Exam #1

Name:

1. [10 pts.] Find the inverse of $f(x) = 5x - 9$, and write it in the form $y = f^{-1}(x)$.

2. [15 pts.] Find all inverses associated with $f(x) = 3/(x^2 + 6)$, and state the domain of each.

3. [10 pts.] Find $(f^{-1})'(3)$ if $f(x) = x^3 + x + 1$.

4. [10 pts.] If $f(x) = \ln(\ln x)$, find $f'(x)$ and give the intervals on which the result is valid.

5. [10 pts.] Find $f'(\pi/4)$ for $f(x) = e^{\sin 2x}$.

6. [10 pts. each] Evaluate each integral

   (a) $\displaystyle\int_0^3 \frac{2x-1}{x+1}\,dx$

   (b) $\displaystyle\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$

   (c) $\displaystyle\int_{-2}^{2} 4^x\,dx$

   (d) $\displaystyle\int \frac{5}{\sqrt{49 - x^2}}\,dx$

7. [10 pts.] Find the derivative using logarithmic differentiation:

$$f(x) = (\tan x)^{\sin x}$$

8. [10 pts. each] Find each derivative.

   (a) $s(t) = \cos(2^t)$

   (b) $f(x) = 4\log_3(x^2 - 1)$

   (c) $g(y) = \cos(\sin^{-1}(2y))$

   (d) $h(z) = \sec^{-1}(\ln z)$

9. [10 pts.] Evaluate using L'Hôpital's Rule:

$$\lim_{x \to 0^+} (1 + x)^{\cot x}$$

10. [10 pts. each] Evaluate each integral via integration by parts.

    (a) $\displaystyle\int x^2 e^{4x}\,dx$

    (b) $\displaystyle\int_0^{\pi/2} x\cos 2x\,dx$

Formulas & Definitions

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan\theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot\theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec\theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc\theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $\dfrac{d}{dx}(b^x) = b^x \ln b$, for $b \in (0,1) \cup (1, \infty)$
6. $\dfrac{d}{dx}\left(\log_b |x|\right) = \dfrac{1}{x \ln b}$, for $x \neq 0$
7. $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
8. $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
9. $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, 1) \cup (1, \infty)$
10. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$
11. $\displaystyle\int b^x\,dx = \frac{1}{\ln b} b^x + C$, for $b \in (0,1) \cup (1, \infty)$
12. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$, for $a \in (0, \infty)$
13. $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$, for $a \neq 0$
14. $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$, for $a \in (0, \infty)$
15. $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
16. $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
17. $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx, \;\; n \neq 1$
18. $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx, \;\; n \neq 1$
19. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$
20. $\displaystyle\int \cot x\,dx = \ln|\sin x| + C$
21. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$