1 Use geometry to evaluate the integral $\int_0^7 \sqrt{49 - t^2}\, dt$.
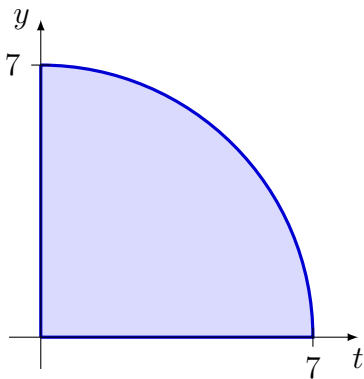
The integrand $y = \sqrt{49 - t^2}$ for $t \in [0, 7]$ represents the top-right quarter of a circle with center at the origin and radius 7 (depicted below). The integral equals the area of this region, which is $\frac{1}{4}\pi(7^2) = \frac{49}{4}\pi$.

2 Let $\int_1^4 f = 6$, $\int_1^4 g = 4$, and $\int_3^4 f = 2$. Evaluate $-\int_4^1 2f$, $\int_1^4 fg$, and $\int_1^3 f$, or state there is not enough information.

We have
$$-\int_4^1 2f = \int_1^4 2f = 2\int_1^4 f = 2(6) = 12,$$
while $\int_1^4 fg$ cannot be evaluated for lack of information, and
$$\int_1^3 f = \int_1^4 f - \int_3^4 f = 6 - 2 = 4.$$

3 Evaluate $\int_4^9 \dfrac{x^2 - 3x}{\sqrt{x}}\, dx$.

By the Fundamental Theorem of Calculus,
$$\int_4^9 \frac{x^2 - 3x}{\sqrt{x}}\, dx = \int_4^9 \left(x^{3/2} - 3x^{1/2}\right) dx = \left[\frac{2}{5}x^{5/2} - 2x^{3/2}\right]_4^9 = \frac{232}{5}.$$

4 Find $\dfrac{d}{dx}\displaystyle\int_{-1}^{\sin x} \dfrac{dt}{\sqrt{t^4 + 12}}$.

By the Fundamental Theorem of Calculus,
$$\frac{d}{dx}\int_{-1}^{\sin x} \frac{dt}{\sqrt{t^4 + 12}} = \frac{1}{\sqrt{\sin^4 x + 12}} \cdot \frac{d}{dx}(\sin x) = \frac{\cos x}{\sqrt{\sin^4 x + 12}}.$$