Math 140 Quiz #3 (Spring 2021)

1 Find the critical points for $f(x) = 4x^{1/2} - x^{5/2}$ on $[0,4]$, then find the absolute maximum and absolute minimum for f on $[0,4]$.

We first note that $f'(x) = 2x^{-1/2} - \frac{5}{2}x^{3/2}$ exists for all $x \in (0,4)$. However, $f'(x) = 0$ does have one solution in $(0,4)$, which is $x = 2/\sqrt{5}$. This is the only critical point. Now we evaluate $f(0) = 0$, $f(4) = -24$, $f(2/\sqrt{5}) \approx 3.026$, and conclude that f has absolute maximum $f(2/\sqrt{5}) \approx 3.026$ and absolute minimum $f(4) = -24$.

2 Determine whether the Mean Value Theorem applies to $f(x) = \dfrac{x}{2x+3}$ on $[-1,2]$. If so, find all points that are guaranteed by the theorem to exist.

Since f is continuous on $[-1,2]$ and differentiable on $(-1,2)$, the Mean Value Theorem does apply. The theorem implies there exists some $c \in (-1,2)$ such that

$$\frac{3}{(2c+3)^2} = f'(c) = \frac{f(2) - f(-1)}{2-(-1)} = \frac{3}{7},$$

which solves to give $c = \frac{-3\pm\sqrt{7}}{2} \approx -2.82, -0.177$. Finally we note that $\frac{-3+\sqrt{7}}{2} \approx -0.177$ lies in $(-1,2)$.

3 Find the intervals on which the function $g(x) = \dfrac{3x}{x^2+2}$ is increasing, and the intervals on which it is decreasing.

First we find that

$$g'(x) = \frac{6-3x^2}{(x^2+2)^2}.$$

By the Increasing/Decreasing Test (also called the Monotonicity Test), g is increasing on any interval I on which $g'(x) > 0$ for all $x \in I$. But $g'(x) > 0$ only holds when $6 - 3x^2 > 0$, or $x^2 < 2$, which has solution set $(-\sqrt{2}, \sqrt{2})$. Next, g is decreasing wherever $g'(x) < 0$, or $x^2 > 2$, which has solution set $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

Therefore g is increasing on $(-\sqrt{2}, \sqrt{2})$, and decreasing on $(-\infty, -\sqrt{2})$ and $(\sqrt{2}, \infty)$.