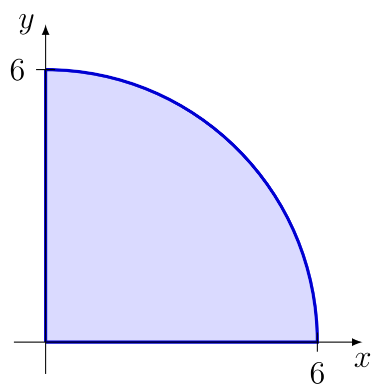
MATH 140 QUIZ #5 (FALL 2020)

1 Use geometry to evaluate the integral $\int_0^6 \sqrt{36 - x^2}\, dx$.

The integrand $y = \sqrt{36 - x^2}$ for $x \in [0, 6]$ represents the top-right quarter of a circle with center at the origin and radius 6 (depicted below). The integral equals the area of this region, which is $\frac{1}{4}\pi(6^2) = 9\pi$.



2 Let $\int_1^4 f = 6$, $\int_1^4 g = 4$, and $\int_3^4 f = 2$. Evaluate $-\int_4^1 2f$, $\int_1^4 fg$, and $\int_1^3 f$, or state there is not enough information.

We have

$$-\int_4^1 2f = \int_1^4 2f = 2\int_1^4 f = 2(6) = 12,$$

while $\int_1^4 fg$ cannot be evaluated for lack of information, and

$$\int_1^3 f = \int_1^4 f - \int_3^4 f = 6 - 2 = 4.$$

3 Evaluate $\displaystyle\int_4^9 \frac{x^2 - 3x}{\sqrt{x}}\, dx$.

By the Fundamental Theorem of Calculus,

$$\int_4^9 \frac{x^2 - 3x}{\sqrt{x}}\, dx = \int_4^9 (x^{3/2} - 3x^{1/2})\, dx = \left[\frac{2}{5}x^{5/2} - 2x^{3/2}\right]_4^9 = \frac{232}{5}.$$

4 Find $\displaystyle\frac{d}{dx}\int_0^{x^3} \frac{dt}{t^2 + 8}$.

By the Fundamental Theorem of Calculus,

$$\frac{d}{dx}\int_0^{x^3} \frac{dt}{t^2 + 8} = \frac{1}{(x^3)^2 + 8} \cdot \frac{d}{dx}(x^3) = \frac{3x^2}{x^6 + 8}.$$