Math 140 Quiz #3 (Fall 2020)

1 Find the absolute extrema of $f(x) = 4x^3 - 21x^2 + 36x$ on the interval $[1, 3]$.

First we find the derivative to be

$$f'(x) = 12x^2 - 42x + 36 = 6(2x - 3)(x - 2),$$

and so $\frac{3}{2}, 2 \in (1, 3)$ are critical points where $f'(x) = 0$. Now, $f(1) = 19$, $f(\frac{3}{2}) = 20.25$, $f(2) = 20$, and $f(3) = 27$, and so there is an absolute minimum at $(1, f(1)) = (1, 19)$ and an absolute maximum at $(3, f(3)) = (3, 27)$.

2 Determine whether Rolle's Theorem applies to $h(x) = \sqrt{x}$ on the interval $[0, a]$ for any $a > 0$. If it does, find all points guaranteed to exist by the theorem. If it doesn't, explain why.

The theorem does not apply since, for any $a > 0$, $h(a) = \sqrt{a} > \sqrt{0} = 0 = h(0)$, and so $h(a) \neq h(0)$.

3 Determine whether the Mean Value Theorem applies to $h(x) = \sqrt{x}$ on the interval $[0, a]$ for any $a > 0$. If it does, find all points guaranteed to exist by the theorem. If it doesn't, explain why.

Function f is continuous on $[1, 3]$ and differentiable on $(1, 3)$, so the Mean Value Theorem applies and states that there is some $1 < c < 3$ such that

$$1 - \frac{1}{c^2} = f'(c) = \frac{f(3) - f(1)}{3 - 1} = \frac{\frac{10}{3} - 2}{2} = \frac{2}{3}.$$

We have $1/c^2 = 1/3$, so $c^2 = 3$, and hence $c = \pm\sqrt{3}$. We now note that $\sqrt{3} \in (1, 3)$.