MATH 140 QUIZ #2 (FALL 2020)

1 Differentiate $f(x) = 4\sqrt{x} - 7x^4 + 9x - 2$.

With the Power Rule,

$$f'(x) = \frac{2}{\sqrt{x}} - 28x^3 + 9.$$

2 Differentiate $g(x) = \dfrac{x^2 - 9}{x^2 + 1}$.

With the Quotient Rule,

$$g'(x) = \frac{(x^2+1)(2x) - (x^2-9)(2x)}{(x^2+1)^2} = \frac{20x}{(x^2+1)^2}.$$

3 Differentiate $H(x) = x^2 \tan x \sin x$.

With the Product Rule,

$$\begin{aligned} H'(x) &= x^2 \tan x \cos x + x^2 \sec^2 x \sin x + 2x \tan x \sin x \\ &= (x + x\sec^2 x + 2\tan x)(x \sin x). \end{aligned}$$

4 Differentiate $y = \dfrac{1 - 2\sin x}{1 + 2\sin x}$.

With the Quotient Rule,

$$y' = \frac{(1+2\sin x)(-2\cos x) - (1-2\sin x)(2\cos x)}{(1+2\sin x)^2} = -\frac{4\cos x}{(1+2\sin x)^2}.$$

5 Evaluate the limit $\displaystyle\lim_{x\to 0} \frac{\tan 7x}{\sin x}$.

Using the limit $\displaystyle\lim_{x\to 0} \frac{\sin x}{x} = 1$, we find that

$$\lim_{x\to 0} \frac{\tan 7x}{\sin x} = \lim_{x\to 0} \left(\frac{1}{\cos 7x} \cdot \frac{\sin 7x}{\sin x} \right) = \underbrace{\lim_{x\to 0} \frac{1}{\cos 7x}}_{1} \cdot \lim_{x\to 0} \frac{\sin 7x}{\sin x}$$

$$= \lim_{x\to 0} \frac{\frac{\sin 7x}{7x}}{\frac{\sin x}{7x}} = \frac{\lim_{x\to 0} \frac{\sin 7x}{7x}}{\frac{1}{7} \lim_{x\to 0} \frac{\sin x}{x}} = \frac{1}{\frac{1}{7} \cdot 1} = 7.$$