1a Evaluate the limit $\lim\limits_{x\to 3} \dfrac{x^2 - 2x - 3}{x - 3}$, or state it does not exist.

$$\lim_{x\to 3} \frac{x^2 - 2x - 3}{x - 3} = \lim_{x\to 3} \frac{(x - 3)(x + 1)}{x - 3} = \lim_{x\to 3}(x + 1) = 4.$$

1b Evaluate the limit $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{16x + 1} - 1}$, or state it does not exist.

$$\lim_{x\to 0}\left(\frac{x}{\sqrt{16x + 1} - 1} \cdot \frac{\sqrt{16x + 1} + 1}{\sqrt{16x + 1} + 1} \right) = \lim_{x\to 0} \frac{\sqrt{16x + 1} + 1}{16} = \frac{\sqrt{1} + 1}{16} = \frac{1}{8}.$$

2a Evaluate the limit $\lim\limits_{x\to -2^+} \dfrac{x - 4}{x(x + 2)}$ as a real number, or as $\pm\infty$, or state it does not exist.

$$\lim_{x\to -2^+} \frac{x - 4}{x(x + 2)} = \frac{-6}{(-2)(\to 0^+)} = +\infty.$$

2b Evaluate the limit $\lim\limits_{x\to -2^-} \dfrac{x - 4}{x(x + 2)}$ as a real number, or as $\pm\infty$, or state it does not exist.

$$\lim_{x\to -2^-} \frac{x - 4}{x(x + 2)} = \frac{-6}{(-2)(\to 0^-)} = -\infty.$$

2c Evaluate the limit $\lim\limits_{x\to -2} \dfrac{x - 4}{x(x + 2)}$ as a real number, or as $\pm\infty$, or state it does not exist.

Does not exist, since the one-sided limits are unequal.

3 Evaluate the limit $\lim\limits_{x\to -\infty} \dfrac{2x}{\sqrt{x^2 - x - 2}}$, or state it does not exist.

For $x \to \infty$ we have $|x| = -x$, and so

$$\lim_{x\to -\infty} \frac{2x}{\sqrt{x^2 - x - 2}} = \lim_{x\to -\infty} \frac{2x}{|x|\sqrt{1 - 1/x - 2/x^2}} = \lim_{x\to -\infty} \frac{-2}{\sqrt{1 - 1/x - 2/x^2}} = -2.$$