MATH 140
SUMMER II 2012
EXAM 3

NAME:

1. Let $f(x) = \sin(x)$.

   (a) [10 pts.] Write the equation of the line L that represents the linear approximation to f at $x = \pi/4$.

   (b) [5 pts.] Use L to estimate $\sin(\pi/4 + 0.1)$

2. [15 pts.] Show, using appropriate theorems and a coherent argument, that the equation $3x - 1 - 2\cos x = 0$ has exactly one real root.

3. [10 pts. each] Determine the indefinite integral, using substitution when necessary.

   (a) $\displaystyle\int (3x^{-2} - 4x^2 + 1)\,dx$

   (b) $\displaystyle\int \left(4\sqrt[3]{t} - \frac{4}{\sqrt{t}}\right) dt$

   (c) $\displaystyle\int \frac{2x^2}{\sqrt{1-4x^3}}\,dx$

4. [10 pts.] The velocity of an object at time t is given by $v(t) = \sin t + 3\cos t$. Find the position of the object at time t, $s(t)$, given that $s(0) = 4$.

5. [10 pts.] Identify f and express the limit as a definite integral on the interval given:

$$\lim_{\Delta\to 0}\sum_{k=1}^{n} 2\bar{x}_k^3 \tan \bar{x}_k\,\Delta x_k\,;\ [-\pi/4, \pi/3]$$

6. [10 pts.] Given that

$$f(x) = \int_0^{\tan x} \cos^3(t)\,dt,$$

find $f'(x)$.

7. [15 pts.] Evaluate

$$\int_2^6 (3x^2 - 5)\,dx$$

using the *definition* of the definite integral.

8. [5 pts. each] Suppose that $\int_2^6 f(x)\,dx = 2$, $\int_2^6 g(x)\,dx = 8$, $\int_5^6 g(x)\,dx = -4$. Evaluate the following.

   (a) $\displaystyle\int_6^2 7f(x)\,dx$

   (b) $\displaystyle\int_2^6 [f(x) - 3g(x)]\,dx$

   (c) $\displaystyle\int_2^5 9g(x)\,dx$

9. [10 pts.] Evaluate each with the Fundamental Theorem of Calculus, using substitution where necessary.

   (a) $\displaystyle\int_1^4 \frac{5t^6 - \sqrt{t}}{t^2}\,dt$

   (b) $\displaystyle\int_0^{\pi/2} \sin^2\theta\cos\theta\,d\theta$

10. [10 pts.] Find the area of the region in the first quadrant bounded by $y = x - 1$ and $y = (x-1)^3$.

11. [10 pts.] Find the area of the region bounded by the curves $x = y(y-1)$ and $x = -y(y-1)$.

12. [10 pts.] Let $\mathcal{R}$ be the region bounded by $y = \sin x$, $y = 1 - \sin x$, $x = \pi/6$, and $x = 5\pi/6$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.