MATH 140
SUMMER II 2012
EXAM 1

NAME:

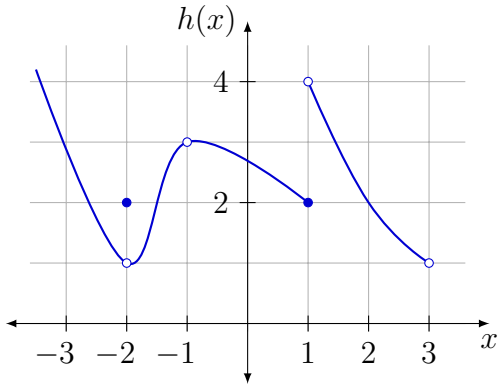
1. [15 pts.] The position of an object is given by $s(t) = 3.2t^2 - 15t + 5$. Find the average velocity of the object over the intervals $[1, 2]$, $[1.9, 2]$, $[1.99, 2]$, and $[h, 2]$, where $h < 2$.

2. [3 pts. each] Use the graph below to find the following limits, if they exist. If a limit does not exist, explain why.

   (a) $\lim_{x\to 1^-} h(x)$ (b) $\lim_{x\to 1^+} h(x)$

   (c) $\lim_{x\to 1} h(x)$ (d) $\lim_{x\to -1} h(x)$

   (e) $\lim_{x\to 2} h(x)$



3. [10 pts. each] Evaluate each limit.

   (a) $\lim_{r\to 3}(r^4 - 7r + 4)^{2/3}$

   (b) $\lim_{t\to -2}\left(\dfrac{t^2}{t+2} + \dfrac{2t}{t+2}\right)$

   (c) $\lim_{x\to 0} \dfrac{\sqrt{2x^2+25} - 5}{x^2}$

4. [10 pts.] Use the definition of limit to prove that

$$\lim_{x\to 5}(3x - 8) = 7.$$

5. [15 pts.] Find all vertical asymptotes $x = a$ of the function

$$f(x) = \frac{x+1}{x^3 - 4x^2 + 4x}.$$

   For each value of a evaluate $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a} f(x)$.

6. [15 pts.] For

$$f(x) = \frac{4x^3}{2x^3 + \sqrt{9x^6 + 15x^4}},$$

   evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to -\infty} f(x)$, and identify any horizontal asymptotes of f.

7. [10 pts.] Show that f is not continuous at 4.

$$f(x) = \begin{cases} x^2 - 5, & \text{if } x \neq 4 \\ 13, & \text{if } x = 4 \end{cases}$$

8. [15 pts.] Let g be given by

$$g(x) = \begin{cases} x^2 + x, & \text{if } x < 1 \\ a, & \text{if } x = 1 \\ 3x + 5, & \text{if } x > 1 \end{cases}$$

   Find the value of a for which g is continuous from the left at 1, and the value of a for which g is continuous from the right at 1. Is there an a value for which g is continuous at 1?

9. Let $f(x) = 3x^2 + 2x - 10$.

   (a) [15 pts.] Use the *definition* of derivative to find f'.

   (b) [5 pts.] Determine an equation for the tangent line to the graph of f at $(1, -5)$.

10. [10 pts. each] Use differentiation rules to find the derivative of each function.

(a) $f(x) = (5x^4 + 3x^2 + 1)(x^3 + 7)$

(b) $g(w) = \dfrac{w^2 - 1}{w^2 + 1}$

11. [15 pts.] Find the points on the graph of

$$f(x) = \frac{x - x^2}{2x^2 + 1}$$

where the slope of the graph is 0.

12. **Extra Credit (15 pts.):** Prove that

$$\lim_{x \to \infty} \frac{5}{x^3} = 0$$

using the appropriate limit definition.