Math 140
Exam #4
Summer II '10

Name:

1. [10 pts.] Estimate the area under the graph of $f(x) = 25 - x^2$ from $x = 0$ to $x = 5$ using five approximating rectangles and right endpoints.

2. [20 pts.] The definite integral can be defined as $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$, where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i\Delta x$. Use this definition to evaluate $\int_{-1}^{3} (1 + x^2)\,dx$.

3. [10 pts.] Evaluate the integral $\int_{-2}^{2} \sqrt{4 - x^2}\,dx$ by interpreting it in terms of areas.

4. [10 pts. each] Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

   (a) $F(x) = \int_x^{10} \sin^4 t\,dt.$

   (b) $y = \int_1^{\cos x} (t + \tan t)\,dt.$

5. [10 pts. each] Evaluate the integral.

   (a) $\int_0^{\pi} (4\sin\theta - 3\cos\theta)\,d\theta$

   (b) $\int_{-4}^{2} (3s - 2|s|)\,ds$

   (c) $\int_1^2 x\sqrt{x-1}\,dx$

   (d) $\int_0^{\sqrt{\pi}} \varphi\cos(\varphi^2)\,d\varphi$

6. [10 pts. each] Find the indefinite integral.

   (a) $\int \cos\theta \sin^6\theta\,d\theta$

   (b) $\int \dfrac{1+4x}{\sqrt{1+x+2x^2}}\,dx$

   (c) $\int \sqrt{\cot x}\csc^2 x\,dx$

7. [15 pts.] Find the area of the region enclosed by the curves $y = \sqrt{x+3}$ and $y = (x+3)/2$.

8. [15 pts.] Find the volume of the solid obtained by rotating about the x-axis the region bounded by the curves $y = x^3$, $y = x$, and $x \geq 0$.

Some Wonderful Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$