MATH 140
SUMMER 2020 NAME:
EXAM 4

1. [5 pts.] Write the following as a definite integral:

$$\lim_{\Delta \to 0} \sum_{j=1}^{n} [9 + 5(x_j^*)^3] \Delta x_j \text{ on } [-7, 4].$$

2. [10 pts. each] Use geometry, rather than Riemann sums, to evaluate each integral.

(a) $\displaystyle\int_{-2}^{3} |y| \, dy$

(b) $\displaystyle\int_{0}^{4} \sqrt{8\xi - \xi^2} \, d\xi$ (Hint: complete the square)

3. [5 pts. each] Suppose $\int_1^4 \varphi(t)dt = -6$, $\int_1^4 \psi(t)dt = 4$, and $\int_3^4 \varphi(t)dt = 2$. Evaluate the following integrals, or state there is not enough information.

(a) $-\displaystyle\int_4^1 -3\varphi(t)dt$, (b) $\displaystyle\int_1^4 \varphi(t)\psi(t)dt$, (c) $\displaystyle\int_1^3 \varphi(t)dt$.

4. [10 pts. each] Evaluate each definite integral using the Fundamental Theorem of Calculus.

(a) $\displaystyle\int_1^4 \frac{w-2}{\sqrt{w}} \, dw$

(b) $\displaystyle\int_{\pi/4}^{\pi/2} \csc^2 \theta \, d\theta$

5. [10 pts.] Simplify the expression: $\displaystyle\frac{d}{dx} \int_{2x}^{0} \frac{dt}{t^2 + \sin t}$.

6. [10 pts. each] Use a change of variables (substitution) to find the following.

(a) $\displaystyle\int \frac{x}{\sqrt{4 - 9x^2}} \, dx$

(b) $\displaystyle\int \sin\alpha \sec^6 \alpha \, d\alpha$

(c) $\displaystyle\int_0^2 2r^3\sqrt{16 - r^4} \, dr$

7. [10 pts.] Find the area of the region bounded by the graphs of $y = 3x - x^2$, $y = x$, and $x = 3$.

8. [10 pts.] What's the volume of the region of space with base consisting of the triangle with corners at $(0, 0)$, $(3, 0)$, and $(0, 3)$, and with cross sections at right angles to the base and parallel to the y-axis that are semicircles? Use a method covered by the homework.

9. [10 pts.] Use an appropriate method to get the volume of the spatial region created by spinning about the x-axis the area enclosed by the graphs of $y = \sqrt{25 - x^2}$, $y = 0$, $x = 2$, and $x = 4$.

10. [10 pts.] Find the length of the graph of

$$C(x) = \frac{(x^2 + 2)^{3/2}}{3}$$

for $-6 \leq x \leq -2$.