MATH 140
SUMMER 2020 NAME:
EXAM 3

1. [10 pts.] Use the Closed Interval Method to find the absolute extreme values of

$$f(x) = 4x^3 - 21x^2 + 36x$$

on $[1, 3]$.

2. [10 pts.] Determine whether the Mean Value Theorem applies to the function

$$f(x) = \frac{x}{x+2}$$

on the interval $[-1, 2]$; if so, find the point(s) that are guaranteed to exist by the Mean Value Theorem.

3. [10 pts.] Find the intervals on which the function

$$f(x) = x^2\sqrt{9 - x^2}$$

is increasing and decreasing, assuming $x \in (-3, 3)$.

4. [7 pts. each] Let $f(x) = \dfrac{12x}{x^2 + 2}$.
   (a) Find the domain and intercepts of f.
   (b) Find the asymptotes of f.
   (c) Find the critical points of f.
   (d) Use the Monotonicity Test to find intervals of increase and decrease, and use either the First Derivative Test or Second Derivative Test to find all local extrema.
   (e) Use the Concavity Test to find intervals where f is concave up or down, and identify inflection points.

5. [10 pts.] The boundary of a field is a right triangle with a straight stream along its hypotenuse and with fences along its other two sides. Find the dimensions of the field with maximum area that can be enclosed using 1000 meters of fencing.

6. [15 pts.] An independent truck driver charges a client \$15 for each hour of driving, plus the cost of fuel. At highway speeds of v miles per hour, the trucker's rig gets $10 - 0.07v$ miles per gallon of diesel fuel. If diesel fuel costs \$2.50 per gallon, what speed v will minimize the cost to the client?

7. [15 pts.] Write the equation of the line that represents the linear approximation to the function $f(x) = \sqrt[4]{x}$ at $a = 81$, then use the approximation to estimate $\sqrt[4]{85}$.

8. [10 pts. each] Use L'Hôpital's Rule, when applicable, to evaluate each limit.

(a) $\lim_{x\to 0^+} \dfrac{x-3\sqrt{x}}{x-\sqrt{x}}$

(b) $\lim_{x\to \pi^-} \dfrac{\csc x + x}{\tan(x/2)}$

(c) $\lim_{x\to\infty} \left(\sqrt{x^2+x} - x\right)$

9. [10 pts. each] Determine the following indefinite integrals.

(a) $\displaystyle\int \frac{y^4 - 2\sqrt{y}+2}{y^2}\, dy.$

(b) $\displaystyle\int \frac{8+\tan\theta}{\sec\theta}\, d\theta.$

10. [10 pts.] Solve the initial value problem:

$$h'(t) = 4(\cos t - \sin t); \quad h(\pi/2) = 0.$$