

MATH 140
SUMMER 2020
EXAM 1

NAME:

1. [3 pts. each] Use the graph below to find the following limits, if they exist. If a limit does not exist, explain why.

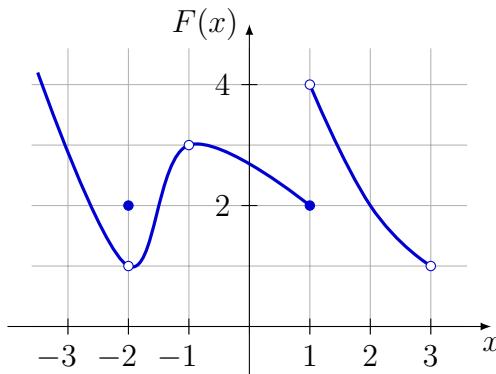
(a) $\lim_{x \rightarrow -1} F(x)$

(b) $\lim_{x \rightarrow -2} F(x)$

(c) $\lim_{x \rightarrow 1^-} F(x)$

(d) $\lim_{x \rightarrow 1^+} F(x)$

(e) $\lim_{x \rightarrow 3^+} F(x)$



2. [10 pts. each] Evaluate each limit algebraically using limit laws, showing work.

(a) $\lim_{x \rightarrow -b} \frac{(x+b)^7 + (x+b)^{10}}{4(x+b)}, \quad b \text{ a fixed real number.}$

(b) $\lim_{h \rightarrow 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$

(c) $\lim_{x \rightarrow 64} \frac{\sqrt{x} - 8}{x - 64}$

(d) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\cos^2 x - 3 \cos x + 2}$

3. [10 pts.] Suppose

$$G(x) = \begin{cases} 3x - 4k, & x < -3 \\ x + 9, & x > -3. \end{cases}$$

Determine a value for k for which the limit $\lim_{x \rightarrow -3} G(x)$ exists, and state the value of the limit.

4. [4 pts. each] Determine the following limits:

$$\lim_{x \rightarrow 3^+} \frac{-5}{(x-3)^3}, \quad \lim_{t \rightarrow -2^-} \frac{t^3 - 5t^2 + 6t}{t^4 - 4t^2}, \quad \lim_{\theta \rightarrow 0^-} \cot 6\theta.$$

5. [10 pts.] Find all vertical asymptotes $x = a$ of the function

$$f(x) = \frac{2x - 1}{2x^3 + 5x^2 - 3x}.$$

For each value of a determine $\lim_{x \rightarrow a^+} f(x)$, $\lim_{x \rightarrow a^-} f(x)$, and $\lim_{x \rightarrow a} f(x)$.

6. [10 pts.] Evaluate the limit

$$\lim_{x \rightarrow -\infty} \frac{8 - 4x^2}{7x^2 + 5x - 12}.$$

7. [15 pts.] Determine $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ for

$$f(x) = \frac{4x^3 + 1}{2x^3 + \sqrt{16x^6 + 1}}.$$

Then give the horizontal asymptotes of f , if any.

8. [10 pts.] Let h be given by

$$h(x) = \begin{cases} 2x^2 - x, & \text{if } x < -1 \\ s, & \text{if } x = -1 \\ 3x - 5, & \text{if } x > -1 \end{cases}$$

Find the value of s for which h is continuous from the left at -1 , and the value of s for which h is continuous from the right at -1 .

9. [10 pts.] Use the precise definition of limit to prove that

$$\lim_{x \rightarrow -2} (4x + 9) = 1.$$

10. Let $f(x) = \sqrt{2x - 1}$.

(a) [10 pts.] Use the limit definition of a derivative to find $f'(1)$.

(b) [5 pts.] Determine an equation for the tangent line to the graph of f at the point $(1, 1)$.

11. [15 pts.] Use the limit definition of a derivative to find $f'(x)$ given that

$$f(x) = \frac{1}{3x - 4}.$$