MATH 140
SUMMER 2019 NAME:
EXAM 4

1. [10 pts.] Use geometry (not Riemann sums) to evaluate the integral $\int_0^6 \sqrt{36 - x^2}\, dx$.

2. [4 pts. each] Suppose $f(x) \geq 0$ on $[0, 2]$, $f(x) \leq 0$ on $[2, 5]$, $\int_0^2 f(x)dx = 6$, and $\int_2^5 f(x)dx = -8$. Evaluate the following:

$$(a) \int_0^5 f(x)dx, \qquad (b) \int_0^5 |f(x)|dx, \qquad (c) \int_2^5 4|f(x)|dx.$$

3. [10 pts. each] Evaluate each definite integral using the Fundamental Theorem of Calculus.

(a) $\int_1^4 \frac{x-2}{\sqrt{x}}\, dx$

(b) $\int_{\pi/4}^{\pi/2} \csc^2\theta\, d\theta$

4. [10 pts.] Simplify the expression: $\frac{d}{dx}\int_x^0 \frac{dp}{p^2+1}$.

5. [10 pts. each] Use a change of variables (substitution) to find the following.

(a) $\int \frac{x}{\sqrt{4-9x^2}}\, dx$

(b) $\int \sin(x)\sec^8(x)\, dx$

(c) $\int_0^2 x^3\sqrt{16-x^4}\, dx$

6. [10 pts.] Find the area of the region enclosed by the curves $y = 3x - x^2$, $y = x$, and $x = 3$.

7. [10 pts.] Use the General Slicing Method to find the volume of the solid whose base is the triangle with vertices $(0, 0)$, $(3, 0)$, and $(0, 3)$, and whose cross sections perpendicular to the base and parallel to the y-axis are semicircles.

8. [10 pts.] Use the Disc Method or Washer Method (whichever is appropriate) to find the volume of the solid generated by revolving about the x-axis the region bounded by the curves $y = \sqrt{25 - x^2}$, $y = 0$, $x = 2$, and $x = 4$.

9. [10 pts.] Use the Shell Method to find the volume of the solid generated by revolving about the y-axis the region bounded by the curves $y = x^3 - x^8 + 1$ and $y = 1$.

10. [10 pts.] Find the length of the curve given by $y = \frac{1}{3}(x^2 + 2)^{3/2}$ for $x \in [1, 2]$.