MATH 140
SUMMER 2019 NAME:
EXAM 1

1. [2 pts. each] For any real number x let $\lfloor x \rfloor$ be the greatest integer less than or equal to x. Evaluate each of the following limits:

$$\lim_{x\to 3^-} \lfloor x \rfloor, \quad \lim_{x\to 3^+} \lfloor x \rfloor, \quad \lim_{x\to -6^-} \lfloor x \rfloor, \quad \lim_{x\to -6^+} \lfloor x \rfloor, \quad \lim_{x\to 0.9} \lfloor x \rfloor.$$

2. [10 pts. each] Evaluate each limit algebraically using limit laws, showing work.

   (a) $\lim_{x\to b} \dfrac{(x-b)^{40} - x + b}{b - x}$, b a fixed real number.

   (b) $\lim_{w\to 1} \left(\dfrac{1}{w^2 - w} - \dfrac{1}{w - 1} \right)$

   (c) $\lim_{x\to 4} \dfrac{3(x-4)\sqrt{x+5}}{3 - \sqrt{x+5}}$

   (d) $\lim_{x\to 0} \dfrac{\cos x - 1}{\sin^2 x}$

3. [10 pts.] Suppose

$$p(x) = \begin{cases} 3x + r, & x < -2 \\ x - 12, & x > -2. \end{cases}$$

   Determine a value for r for which the limit $\lim_{x\to -2} p(x)$ exists, and state the value of the limit.

4. [4 pts. each] Determine the following limits:

$$\lim_{x\to 3^-} \frac{-5}{(x-3)^3}, \quad \lim_{t\to -2^+} \frac{t^3 - 5t^2 + 6t}{t^4 - 4t^2}, \quad \lim_{\theta\to 0^-} \csc\theta.$$

5. [10 pts.] Find all vertical asymptotes $x = a$ of the function

$$f(x) = \frac{x+1}{x^3 - 4x^2 + 4x}.$$

   For each value of a determine $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, and $\lim_{x\to a} f(x)$.

6. [10 pts.] Evaluate the limit

$$\lim_{x\to\infty} \frac{4x^2 - 7}{8x^2 + 5x + 2}.$$

7. [15 pts.] Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to -\infty} f(x)$ for

$$f(x) = \frac{\sqrt{x^2+1}}{2x+1}.$$

Then give the horizontal asymptotes of f, if any.

8. [10 pts.] Let g be given by

$$g(x) = \begin{cases} x^2 - 2x, & \text{if } x < 1 \\ a, & \text{if } x = 1 \\ 3x + 9, & \text{if } x > 1 \end{cases}$$

Find the value of a for which g is continuous from the left at 1, and the value of a for which g is continuous from the right at 1.

9. [10 pts.] Use the precise definition of limit to prove that

$$\lim_{x \to 8}(4x - 5) = 27.$$

10. Let $f(x) = \sqrt{x + 3}$.

    (a) [10 pts.] Use the limit definition of a derivative to find $f'(1)$.

    (b) [5 pts.] Determine an equation for the tangent line to the graph of f at the point $(1, 2)$.

11. [15 pts.] Use the limit definition of a derivative to find $f'(x)$ given that

$$f(x) = \frac{1}{x + 2}.$$