MATH 140
SUMMER 2018 NAME:
EXAM 1

1. $\boxed{\text{10 pts.}}$ Use the precise definition of limit to prove that

$$\lim_{x \to -3} (4x + 10) = -2.$$

2. $\boxed{\text{3 pts. each}}$ Let

$$f(x) = \begin{cases} 1, & x \leq -5 \\ \sqrt{25 - x^2}, & -5 < x < 5 \\ 2x - 10, & x \geq 5. \end{cases}$$

Compute each limit, if it exists.

(a) $\lim_{x \to -5} f(x)$ (b) $\lim_{x \to 5^-} f(x)$ (c) $\lim_{x \to 5^+} f(x)$ (d) $\lim_{x \to 5} f(x)$ (e) $\lim_{x \to 3} f(x)$

3. $\boxed{\text{10 pts. each}}$ Evaluate each limit algebraically using limit laws, showing work.

(a) $\displaystyle\lim_{t \to 2} \frac{3t^2 - 7t + 2}{2 - t}$.

(b) $\displaystyle\lim_{x \to 49} \frac{\sqrt{x} - 7}{x - 49}$.

4. $\boxed{\text{10 pts.}}$ Suppose

$$h(x) = \begin{cases} b - 3x, & x \leq 2 \\ x + 2, & x > 2. \end{cases}$$

Determine a value for b for which the limit $\lim_{x \to 2} h(x)$ exists, and state the value of the limit.

5. $\boxed{\text{10 pts.}}$ Find all vertical asymptotes $x = a$ of the function

$$f(x) = \frac{x^2 - 9x + 14}{x^2 - 5x + 6}.$$

For each value of a determine $\lim_{x \to a^+} f(x)$, $\lim_{x \to a^-} f(x)$, and $\lim_{x \to a} f(x)$.

6. $\boxed{\text{10 pts.}}$ Evaluate the limit

$$\lim_{x \to \infty} \frac{7 - 4x^2}{6x^2 + 5x + 2}.$$

7. $\boxed{\text{15 pts.}}$ Determine $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ for

$$f(x) = \frac{\sqrt{x^2 + 1}}{2x + 1}.$$

Then give the horizontal asymptotes of f, if any.

8. $\boxed{\text{10 pts.}}$ Show that f is not continuous at 4.

$$f(x) = \begin{cases} x^2 - 5, & \text{if } x \neq 4 \\ 13, & \text{if } x = 4 \end{cases}$$

9. $\boxed{\text{10 pts.}}$ Let g be given by

$$g(x) = \begin{cases} x^2 + x, & \text{if } x < 1 \\ a, & \text{if } x = 1 \\ 3x + 5, & \text{if } x > 1 \end{cases}$$

Find the value of a for which g is continuous from the left at 1, and the value of a for which g is continuous from the right at 1. Is there an a value for which g is continuous at 1?

10. Let $f(x) = \sqrt{x}$.

(a) $\boxed{\text{15 pts.}}$ Use the definition of derivative to find $f'(4)$.

(b) $\boxed{\text{5 pts.}}$ Determine an equation for the tangent line to the graph of f at the point $(4, 2)$.