MATH 140
SUMMER 2014 NAME:
EXAM 3

1. $\boxed{\text{10 pts.}}$ Does there exist a continuous function f such that $f(-3) = 11$, $f(9) = -5$, and $f'(x) \geq -1$ for all $x \in (-3, 9)$? Either produce such a function, or prove that there can be no such function using appropriate theorems.

2. $\boxed{\text{15 pts.}}$ Prove that
$$3x - 1 - 2\cos x = 0$$
must have exactly one real root using appropriate theorems and a coherent argument.

3. $\boxed{\text{10 pts.}}$ Find the solution to the initial value problem:
$$p'(t) = \frac{1}{2\sqrt{t}}, \quad p(4) = 6.$$

4. $\boxed{\text{10 pts. each}}$ Determine the indefinite integral, using substitution wherever necessary.

(a) $\displaystyle\int \left(\sqrt[4]{x^3} + \sqrt{x^5}\right) dx$

(b) $\displaystyle\int \sec(5y)\tan(5y)dy$

(c) $\displaystyle\int \frac{x}{\sqrt{4 - 9x^2}} dx$

5. $\boxed{\text{15 pts.}}$ Evaluate
$$\int_1^5 (4x - 3)\, dx$$
using the *definition* of the definite integral,
$$\int_a^b f = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(\bar{x}_k)\Delta x_k.$$
Possibly useful formulas:
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \text{and} \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

6. $\boxed{\text{5 pts. each}}$ Suppose that $\int_2^6 f(x)\, dx = -4$, $\int_2^6 g(x)\, dx = 7$, $\int_5^6 g(x)\, dx = 20$. Evaluate the following.

(a) $\displaystyle\int_6^2 7f(x)\, dx$

(b) $\displaystyle\int_2^6 [f(x) - 3g(x)]\, dx$

(c) $\displaystyle\int_2^5 9g(x)\, dx$

7. $\boxed{\text{10 pts.}}$ Given that

$$\Psi(x) = \int_0^{\tan x} t^2 \cos^9(6-t)dt,$$

find $\Psi'(x)$.

8. $\boxed{\text{10 pts. each}}$ Evaluate each with the Fundamental Theorem of Calculus, using substitution where necessary.

   (a) $\displaystyle\int_1^9 \frac{3x^6 - 2\sqrt{x}}{x^2} dx$

   (b) $\displaystyle\int_0^{\pi/4} \cos^2\theta \sin\theta \, d\theta$

9. $\boxed{\text{10 pts.}}$ Find the area of the region in the first quadrant bounded by $y = x - 1$ and $y = (x-1)^3$.

10. $\boxed{\text{10 pts.}}$ Use the General Slicing Method to find the volume of the solid having a semicircular base of radius 5 whose cross sections perpendicular to the base and parallel to the diameter are squares.

11. $\boxed{\text{10 pts.}}$ Let $\mathcal{R}$ be the region bounded by

$$y = \frac{1}{\sqrt[4]{1-x}},$$

$y = 0$, $x = 0$, and $x = \frac{1}{2}$. Use the Disc Method to find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.