MATH 140
SUMMER 2014 NAME:
EXAM 2

1. [10 pts. each] Find the derivative of the function.
   (a) $y = (8t - 3t^2)^{16}$
   (b) $y = \sin^6\theta + \cos^6\theta$
   (c) $f(x) = \sqrt{x - \sqrt{x}}$

2. [10 pts. each] Let
$$x^4 + 2x^2y^2 + y^4 = \frac{25}{4}xy^2.$$
   (a) Use implicit differentiation to find dy/dx.
   (b) Determine an equation of the tangent line to the curve at the point $(1, 2)$.

3. [10 pts.] The height of a triangle is decreasing at a rate of 1 cm/min while the area is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the height is 12 cm and the area is 150 cm^2?

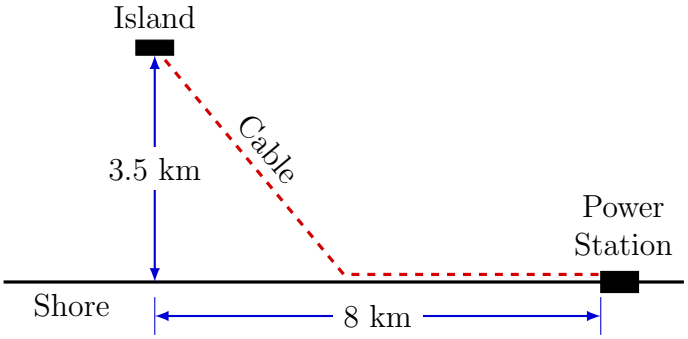
4. [10 pts.] A 13-ft ladder is leaning against a vertical wall when Vladimir begins pulling the foot of the ladder away from the wall at a rate of 0.5 ft/s. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 ft from the wall?

5. [10 pts.] Find the critical points of
$$f(x) = x^3 - 2x^2 - 5x + 6$$
on the interval $I = [4, 8]$, then determine the global extrema of f on I.

6. Let $f(x) = \dfrac{3x - 5}{x^2 - 1}$.
   (a) [5 pts.] Find the domain and intercepts of f.
   (b) [5 pts.] Find the asymptotes of f.
   (c) [10 pts.] Use the Monotonicity Test to find intervals of increase and decrease, then find critical points and use the First Derivative Test to find all local extrema.
   (d) [10 pts.] Use the Concavity Test to find intervals where f is concave up or down, and identify inflection points.
   (e) [5 pts.] Sketch the graph of f.

7. [10 pts.] A 216 m^2 rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

8. [15 pts.] An island is 3.5 km from the nearest point on a straight shoreline, and that point is 8 km from a power station. A utility company plans to lay electrical cable underwater from the island to the shore and then underground along the shore to the power station. Assume that it costs \$2400 per kilometer to lay underwater cable and \$1200 per kilometer to lay underground cable. At what point should the underwater cable meet the shore in order to minimize the cost of the project?



9. Let $f(x) = \sqrt[5]{x}$.

   (a) [10 pts.] Find the linear function L that provides a linear approximation of f at $x = 32$.

   (b) [5 pts.] Use L to estimate $\sqrt[5]{33}$, rounding to the millionths place.