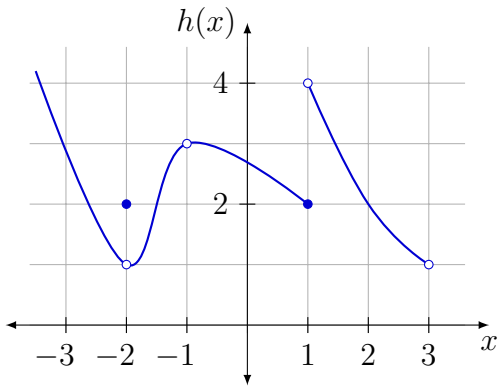
MATH 140
SUMMER 2014 NAME:
EXAM 1

1. [3 pts. each] Use the graph below to find the following limits, if they exist. If a limit does not exist, explain why.

(a) $\lim_{x\to 1^-} h(x)$ (b) $\lim_{x\to 1^+} h(x)$ (c) $\lim_{x\to 1} h(x)$ (d) $\lim_{x\to 3^-} h(x)$ (e) $\lim_{x\to -2} h(x)$



2. [10 pts. each] Evaluate each limit.

(a) $\lim_{r\to 3}(r^4 - 7r + 4)^{2/3}$

(b) $\lim_{t\to -2}\left(\dfrac{t^2}{t+2} + \dfrac{2t}{t+2}\right)$

(c) $\lim_{x\to 0} \dfrac{\sqrt{2x^2+25} - 5}{x^2}$

3. [10 pts.] Suppose the function g satisfies the inequality

$$x^2 - 5x - 2\cos x \le g(x) \le \sin x - 2$$

for all $x \in (0, 1)$. Use the Squeeze Theorem to evaluate $\lim_{x\to 0^+} g(x)$.

4. [15 pts.] For

$$f(x) = \frac{4x^3}{2x^3 + \sqrt{9x^6 + 15x^4}},$$

evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to -\infty} f(x)$, and identify any horizontal asymptotes of f.

5. [10 pts.] Show that f is not continuous at 4.

$$f(x) = \begin{cases} x^2 - 5, & \text{if } x \neq 4 \\ 13, & \text{if } x = 4 \end{cases}$$

6. [15 pts.] Let g be given by

$$g(x) = \begin{cases} x^2 + x, & \text{if } x < 1 \\ a, & \text{if } x = 1 \\ 3x + 5, & \text{if } x > 1 \end{cases}$$

Find the value of a for which g is continuous from the left at 1, and the value of a for which g is continuous from the right at 1. Is there an a value for which g is continuous at 1?

7. Let $f(x) = \sqrt{3x + 1}$.

   (a) [15 pts.] Use the definition of derivative to find $f'(1)$.

   (b) [5 pts.] Determine an equation for the tangent line to the graph of f at $(1, 2)$.

8. [10 pts. each] Use differentiation rules to find the derivative of each function.

   (a) $f(x) = (5x^4 + 3x^2 + 1)(x^3 + 7)$

   (b) $g(t) = \dfrac{t^2 - 1}{t^2 + 1}$

   (c) $y = \sin x \tan x$

   (d) $y = \dfrac{2 \cos x}{1 + \sin x}$

9. [5 pts. each] The position (in meters) of an object at time t (in seconds) is given by

$$s(t) = 2t^3 - 21t^2 + 60; \quad 0 \leq t \leq 6.$$

   (a) Find the object's velocity function. When is the object at rest?

   (b) Find the object's acceleration function. When is the object's acceleration positive, and when is it negative?