MATH 140
SUMMER 2012
EXAM 3

NAME:

1. Let $f(x) = \sqrt[5]{x}$.

   (a) $\boxed{\text{10 pts.}}$ Write the equation of the line L that represents the linear approximation to f at $x = 32$.

   (b) $\boxed{\text{5 pts.}}$ Use L to estimate $\sqrt[5]{33}$

2. $\boxed{\text{15 pts.}}$ Show, using appropriate theorems,[1] that the equation $2x - 1 - \sin x = 0$ has exactly one real root.

3. $\boxed{\text{10 pts. each}}$ Determine the following indefinite integrals, using substitution when necessary.

   (a) $\displaystyle\int (3x^{-2} - 4x^2 + 1)dx$

   (b) $\displaystyle\int [\cos(4t) - \sin(t/4)]dt$

   (c) $\displaystyle\int \frac{2x^2}{\sqrt{1-4x^3}}dx$

4. $\boxed{\text{10 pts.}}$ Find the solution to the initial value problem $f'(x) = 8x - 5;\ \ f(0) = 4$.

5. $\boxed{\text{10 pts.}}$ The velocity of an object (in meters per second) is given by $v(t) = 1/(2t+1)$. Approximate the object's displacement during the time interval $0 \le t \le 10$ by subdividing the interval into 5 subintervals, using the midpoint of each subinterval to compute the height of the rectangles.

6. $\boxed{\text{10 pts.}}$ Identify f and express the limit as a definite integral on the interval given:

$$\lim_{\Delta \to 0} \sum_{k=1}^{n} \bar{x}_k \cos \bar{x}_k \, \Delta x_k\,;\ [1, 2].$$

7. $\boxed{\text{5 pts. each}}$ Suppose that $\int_0^3 f(x)dx = 2$, $\int_3^6 f(x)dx = -9$ and $\int_3^6 g(x)dx = 5$. Evaluate the following.

   (a) $\displaystyle\int_0^3 5f(x)dx$

   (b) $\displaystyle\int_3^6 [3f(x) - g(x)]dx$

   (c) $\displaystyle\int_6^3 [f(x) + 2g(x)]dx$

8. $\boxed{\text{15 pts.}}$ Use the *definition* of the definite integral to evaluate $\displaystyle\int_2^6 (3x^2 - 5)dx$

9. $\boxed{\text{10 pts.}}$ Simplify $\displaystyle\frac{d}{dx}\int_7^{x^4} \sin^5(t)dt$

10. $\boxed{\text{10 pts.}}$ Evaluate each with the Fundamental Theorem of Calculus, using substitution where necessary.

    (a) $\displaystyle\int_1^4 \frac{5t^6 - \sqrt{t}}{t^2}dt$

    (b) $\displaystyle\int_0^{\pi/2} \sin^2\theta \cos\theta \, d\theta$

11. $\boxed{\text{10 pts.}}$ Find the area of the region bounded by the curves $f(x) = 2x^2$ and $g(x) = x^2 + 4$

12. $\boxed{\text{10 pts.}}$ Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = 4 - x^2$, $y = 0$, and $x = 0$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.

[1] This means doodling a graph and pointing and hooting at it will not suffice.