MATH 140
SUMMER 2012
EXAM 2

NAME:

1. [10 pts.] Evaluate the limit:[1] $\lim_{x\to 0} \dfrac{\tan 6x}{x}$

2. [10 pts.] Find the derivative of
$$f(x) = \frac{\sin x}{2 - \tan x}$$

3. [10 pts.] Find an equation of the tangent line to the curve $y = \csc x$ at $x = \pi/4$.

4. [10 pts. each] Find the derivative.
   (a) $f(x) = (4x^3 - 9)^{10}$
   (b) $g(t) = \sin(4\cos t)$
   (c) $h(x) = \sqrt{x + \sqrt{x}}$

5. [10 pts.] Use implicit differentiation to find y', given that $\cos(y^2) + 2x = y^3$.

6. [10 pts.] Find an equation of the tangent line to the curve $x^4 = 2x^2 + 2y^2$ at the point $(2, 2)$.

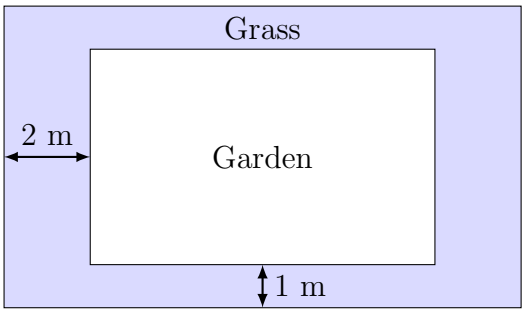
7. [10 pts.] A rectangle initially has dimensions 2 cm by 4 cm. All sides begin increasing in length at a rate of 1 cm/s. At what rate is the area of the rectangle increasing after 20 seconds?

8. [10 pts.] A rope passing through a capstan on a dock is tied to a boat offshore. If the capstan is 5 ft above the water and Popeye pulls the rope in at a constant rate of 3 ft/s, how fast is the boat traveling when it is 10 ft from the deck?

9. [10 pts.] Find the critical points of
$$f(x) = x^3 - 2x^2 - 5x + 6$$
on the interval $I = [4, 8]$, then determine the global extrema of f on I.

10. Let $f(x) = \dfrac{x^3 - 1}{x^3 + 1}$.
    (a) [5 pts.] Find the domain of f.
    (b) [5 pts.] Find the intercepts of f.
    (c) [10 pts.] Use limits to find the asymptotes of f.
    (d) [10 pts.] Use f' to find intervals of increase or decrease, then get critical points and use the First Derivative Test to find local extrema.
    (e) [10 pts.] Use f'' and the Concavity Test to find intervals where f is concave up or down, and identify inflection points.
    (f) [5 pts.] Sketch the graph of f.

11. [10 pts.] A rectangular garden with an area of 30 m^2 is surrounded by a grass border 1 m wide on two sides and 2 m wide on the other two sides. What dimensions of the garden minimize the combined area of the garden and borders?



12. [15 pts.] A square-based, box-shaped shipping crate is designed to have a volume of 16 ft^3. The material used to make the base costs twice as much (per ft^2) as the material in the sides, and the material used to make the top costs half as much (per ft^2) as the material in the sides. What are the dimensions of crate that minimize the cost of materials?

[1] This limit must be done analytically, and not by "guessing" using tables of numbers.