Math 140
Exam #2
Summer 2011

Name:

1. [10 pts.] Evaluate $\lim_{x \to 0} \dfrac{\tan 9x}{\sin x}$

2. [10 pts.] Find the derivative of $f(x) = \dfrac{x \sin x}{1 + \cos x}$

3. [10 pts.] Find an equation of the tangent line to the curve $y = 4 \sin x \cos x$ at $x = \pi/3$.

4. [10 pts. each] Find the derivative of each function.
   (a) $f(x) = \tan(4x^7)$
   (b) $g(x) = 5(3x^8 + x)^{-4}$
   (c) $h(x) = \sqrt{x + \sqrt{x}}$

5. [10 pts.] Use implicit differentiation to find dy/dx, given that $(xy + 1)^3 = x - y^2 + 8$.

6. [10 pts.] Find an equation of the tangent line to the curve $x^3 + y^3 = 2xy$ at the point $(1, 1)$.

7. [10 pts.] A spherical balloon is inflated and its volume increases at a rate of 35 cm^3/min. What is the rate of change of its radius when the radius is 20 cm?

8. [10 pts.] A 16-foot ladder is leaning against a wall when Archimedes begins pulling the base of the ladder away from the wall at a rate of 0.7 ft/sec. How fast is the top of the ladder sliding down the wall when the base of the ladder is 10 ft from the wall?

9. [10 pts.] Find the critical points of the function $f(x) = (4x - 3)/x^2$ on the interval $[1, 4]$, then determine the absolute extreme values of f on this interval.

10. [10 pts. each] Let $g(x) = 200 + 8x^3 + x^4$.
    (a) Use the First Derivative Test to find the intervals of increase and decrease, and any local maximum and minimum values.
    (b) Use the Concavity Test to find the intervals of concavity and any inflection points.

11. [10 pts.] What two positive real numbers whose product is 50 have the smallest possible sum?

12. [15 pts.] A square-based, box-shaped shipping crate is designed to have a volume of 16 ft^3. The material used to make the base costs twice as much (per ft^2) as the material in the sides, and the material used to make the top costs half as much (per ft^2) as the material in the sides. What are the dimensions of crate that minimize the cost of materials?