Math 140
Exam #1
Summer 2011

Name:

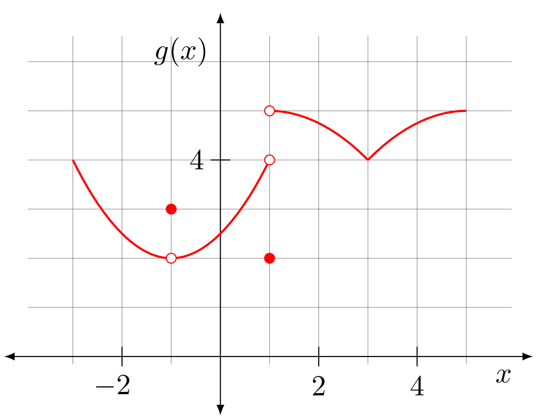
1. [15 pts.] The position of an object moving along a line is given by $s(t) = -4.9t^2 + 30t + 20$. Find the average velocity of the object over the following intervals: $[0,3]$, $[0,2]$, $[0,1]$, and $[0,h]$ (where $h > 0$ is a real number).

2. [3 pts. each] Use the graph below to find the following limits, if they exist. If a limit does not exist, explain why.

   (a) $\lim_{x\to -1} g(x)$ (b) $\lim_{x\to 1^-} g(x)$

   (c) $\lim_{x\to 1^+} g(x)$ (d) $\lim_{x\to 1} g(x)$

   (e) $\lim_{x\to 5^-} g(x)$



3. [10 pts. each] Evaluate each limit.

   (a) $\lim_{x\to 3^-} \sqrt{\dfrac{x-3}{2-x}}$

   (b) $\lim_{x\to 3} \dfrac{x^2-2x-3}{x-3}$

   (c) $\lim_{x\to 25} \dfrac{\sqrt{x}-5}{x-25}$

4. [10 pts.] Evaluate each limit. If a limit does not exist, state whether it equals $+\infty$ or $-\infty$, where possible; otherwise answer with D.N.E.

$$\lim_{x\to 7^+} \frac{9}{x-7}, \quad \lim_{x\to 7^-} \frac{9}{x-7}, \quad \lim_{x\to 7} \frac{9}{x-7}$$

5. [10 pts.] Prove that $\lim_{x\to 0} x^4 \cos\dfrac{2}{x} = 0$ using the Squeeze Theorem.

6. [15 pts.] Given that $f(x) = \dfrac{\sqrt{x^2+2x+6}-3}{x-1}$, evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to -\infty} f(x)$, and identify any horizontal asymptotes of f. Also find the vertical asymptotes of f, if any.

7. [10 pts.] Show that f is not continuous at 1.

$$f(x) = \begin{cases} 1-x^2, & \text{if } x < 1 \\ 1/x, & \text{if } x \geq 1 \end{cases}$$

8. [10 pts.] Use the Intermediate Value Theorem to show that the equation $2x^3 + x - 2 = 0$ has at least one solution in the interval $(-1,1)$.

9. [10 pts. each] Let $f(x) = \dfrac{1}{3-2x}$.

   (a) Find $f'(-1)$ using the limit definition of derivative.

   (b) Determine an equation of the tangent line to the graph of f at $\left(-1, \frac{1}{5}\right)$

10. [10 pts. each] Find the derivative of each using differentiation rules.

   (a) $g(w) = 3w^{-9}$

   (b) $f(t) = 6t^8 - t^3$

   (c) $h(x) = \dfrac{x^4+1}{x^2-1}$