

Math 140
Exam #1
Summer '10

Name:

1. 10 pts. each Evaluate the limit. If a limit does not equal a real number, state whether it equals $-\infty$ or $+\infty$ where appropriate, otherwise answer with "DNE" (Does Not Exist).

(a) $\lim_{x \rightarrow -2^+} \frac{x-1}{x^2(x+2)}$

(b) $\lim_{x \rightarrow -1} \frac{x^2 - 4x}{x^2 - 3x - 4}$

(c) $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7}$

(d) $\lim_{x \rightarrow 3} (2x + |x-3|)$

2. 10 pts. If $2x \leq g(x) \leq x^4 - x^2 + 2$ for all x , evaluate $\lim_{x \rightarrow 1} g(x)$.

3. 10 pts. Prove that $\lim_{x \rightarrow 0} x^4 \cos \frac{2}{x} = 0$ using the Squeeze Theorem.

4. 15 pts. each Prove the statement using the ϵ - δ definition of limit.

(a) $\lim_{x \rightarrow 3} (2x + 7) = 13$

(b) $\lim_{x \rightarrow 2} (5 - 7x) = -9$

5. 10 pts. Explain why the function is discontinuous at 1.

$$f(x) = \begin{cases} 1 - x^2, & \text{if } x < 1 \\ 1/x, & \text{if } x \geq 1 \end{cases}$$

6. 15 pts. Find the numbers at which φ is discontinuous. At which of these numbers is φ continuous from the right, from the left, or neither?

$$\varphi(x) = \begin{cases} x + 1, & \text{if } x \leq 1 \\ 1/x, & \text{if } 1 < x < 3 \\ \sqrt{x-3}, & \text{if } x \geq 3 \end{cases}$$

7. 15 pts. Use the Intermediate Value Theorem to prove that the equation $\sqrt{x-5} = \frac{1}{x+3}$ has at least one real root.

8. 15 pts. Find an equation of the tangent line to the curve $y = \frac{x-1}{x-2}$ at the point $(3, 2)$.

9. 15 pts. If a ball is thrown into the air with an initial velocity of 40 ft/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$.