MATH 140
SPRING 2024 NAME:
EXAM 4

1. [10 pts. each] Use geometry (not Riemann sums) to evaluate the definite integral.

   (a) $\displaystyle\int_{-2}^{14} |x-3|\,dx$

   (b) $\displaystyle\int_0^4 f(x)\,dx$, where $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ 3x-1, & \text{if } x > 2 \end{cases}$

2. [10 pts. each] Evaluate each definite integral using the Fundamental Theorem of Calculus.

   (a) $\displaystyle\int_0^4 t(t-2)(t-4)\,dt$

   (b) $\displaystyle\int_1^8 2\sqrt[3]{x}\,dx$

   (c) $\displaystyle\int_{\pi/4}^{3\pi/4} (\cot^2\theta + 1)\,d\theta$

3. [10 pts.] Find the derivative: $\displaystyle\frac{d}{dx}\int_{\cos x}^{9} \frac{6}{\sqrt{t^6+9}}\,dt.$

4. [10 pts. each] Use a change of variables (substitution) to find the following.

   (a) $\displaystyle\int_0^2 \frac{2x}{(x^2+1)^2}\,dx$

   (b) $\displaystyle\int \sec^2(10x+7)\,dx$

   (c) $\displaystyle\int (z+1)\sqrt{3z+2}\,dz$

5. [10 pts.] Find the area of the region bounded by the curves $y = x/4$ and $x = y^3$.

6. [10 pts.] Use the General Slicing Method to find the volume of the solid whose base is the triangle with vertices $(0,0)$, $(3,0)$, and $(0,3)$, and whose cross sections perpendicular to the base and parallel to the y-axis are semicircles.

7. [10 pts.] Use the Disc Method or Washer Method (whichever is appropriate) to find the volume of the solid generated by revolving about the x-axis the region bounded by the curves $y = \sqrt{25-x^2}$, $y = 0$, $x = 2$, and $x = 4$.