MATH 140
SPRING 2024 NAME:
EXAM 1

1. [10 pts. each] Evaluate each limit analytically using limit laws, showing work.

   (a) $\lim_{t\to -1} \dfrac{(2t-1)^2 - 9}{t+1}$

   (b) $\lim_{x\to 2}\left(\dfrac{1}{x-2} - \dfrac{2}{x^2-2x}\right)$

   (c) $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3}$

   (d) $\lim_{\theta\to 0} \dfrac{\cos\theta - 1}{\cos^2\theta - 1}$

2. [5 pts. each] Determine each limit to be $-\infty$, $+\infty$, a real number, or show that all that can be said is that it does not exist.

   (a) $\lim_{y\to 1} \dfrac{2y}{|1-y|}$

   (b) $\lim_{z\to 3^-} \dfrac{z^2-3z+2}{z-3}$

   (c) $\lim_{x\to 2^+} \dfrac{1}{\sqrt{x(x-2)}}$

   (d) $\lim_{x\to 2^-} \dfrac{1}{\sqrt{x(x-2)}}$

   (e) $\lim_{x\to -\infty} (3x^7 + x^2)$

3. [15 pts. each] Let $f(x) = \dfrac{\sqrt{x^2+2x+6}-3}{x-1}$.

   (a) Evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to -\infty} f(x)$, and then identify any horizontal asymptotes of f.

   (b) Find the vertical asymptotes of f. For each vertical asymptote $x = a$ determine $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$.

4. Let $F(x) = \begin{cases} x^3 + 4x + 1, & \text{if } x \le 0 \\ 2x^3, & \text{if } x > 0 \end{cases}$

   (a) [10 pts.] Prove or disprove that F is continuous at 0.

   (b) [5 pts.] Is F continuous from the left or the right at 0? Both? Neither?

   (c) [5 pts.] On what intervals is F continuous?

5. Let $g(x) = \dfrac{1}{3x-1}$.

(a) [10 pts.] Use the limit definition of a derivative to find $g'(2)$.

(b) [5 pts.] Determine an equation for the tangent line to the graph of g at the point $(2, \frac{1}{5})$.