Math 140 Spring 2022 Exam 1

NAME:

- 1. <u>3 pts. each</u> Use the graph below to find the following limits, if they exist. If a limit does not exist, explain why.
 - (a) $\lim_{x \to -1} F(x)$ (b) $\lim_{x \to -2} F(x)$ (c) $\lim_{x \to 1^-} F(x)$ (d) $\lim_{x \to 1^+} F(x)$ (e) $\lim_{x \to 3^+} F(x)$ $F(x) + \frac{1}{4} + \frac{1$

2. 10 pts. each Evaluate each limit algebraically using limit laws, showing work.

(a) $\lim_{x \to -b} \frac{(x+b)^7 + (x+b)^{10}}{4(x+b)}$, *b* a fixed real number. (b) $\lim_{h \to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$ (c) $\lim_{x \to 81} \frac{\sqrt{x} - 9}{x - 81}$ (d) $\lim_{x \to 0} \frac{1 - \cos x}{\cos^2 x - 3\cos x + 2}$

3. 10 pts. Suppose

$$G(x) = \begin{cases} 3x + 2k, & x < -3\\ x + 9, & x > -3. \end{cases}$$

Determine a value for k for which the limit $\lim_{x\to -3} G(x)$ exists, and state the value of the limit.

4. 4 pts. each Determine the following limits:

$$\lim_{x \to 3^+} \frac{-5}{(x-3)^3}, \quad \lim_{t \to -2^-} \frac{t^3 - 5t^2 + 6t}{t^4 - 4t^2}, \quad \lim_{\theta \to 0^-} \cot 6\theta.$$

5. 10 pts. Find all vertical asymptotes x = a of the function

$$f(x) = \frac{x+1}{x^3 - 4x^2 + 4x}.$$

For each value of a determine $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, and $\lim_{x\to a} f(x)$.

6. 10 pts. Evaluate the limit

$$\lim_{x \to -\infty} \frac{8 - 4x^2}{7x^2 + 5x - 12}$$

7. 15 pts. Let

$$f(x) = \frac{4x^3 + 1}{2x^3 + \sqrt{16x^6 + 1}}.$$

- (a) 10 pts. Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
- (b) 5 pts. Find the horizontal asymptotes of f, if any.
- 8. 10 pts. Let h be given by

$$h(x) = \begin{cases} 2x^2 + x, & \text{if } x < -1 \\ s, & \text{if } x = -1 \\ 4 - 3x, & \text{if } x > -1 \end{cases}$$

Find the value of s for which h is continuous from the left at -1, and the value of s for which h is continuous from the right at -1.

- 9. Let $f(x) = \sqrt{x-3}$.
 - (a) 10 pts. Use the limit definition of a derivative to find f'(4).
 - (b) 5 pts. Determine an equation for the tangent line to the graph of f at the point (4, 1).