Name:
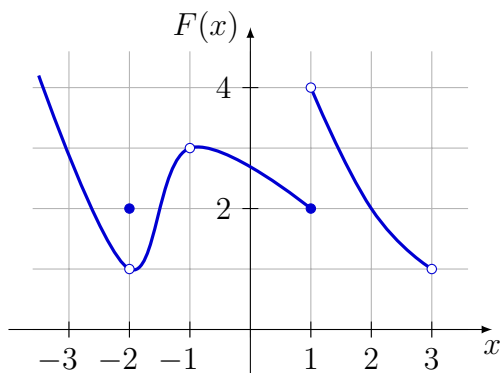
1. 3 pts. each Use the graph below to find the following limits, if they exist. If a limit does not exist, explain why.

(a) $\lim\limits_{x \to -1} F(x)$ (b) $\lim\limits_{x \to -2} F(x)$ (c) $\lim\limits_{x \to 1^-} F(x)$ (d) $\lim\limits_{x \to 1^+} F(x)$ (e) $\lim\limits_{x \to 3^+} F(x)$

2. 10 pts. each Evaluate each limit algebraically using limit laws, showing work.

(a) $\lim\limits_{x \to -b} \dfrac{(x + b)^7 + (x + b)^{10}}{4(x + b)}$, b a fixed real number.

(b) $\lim\limits_{h \to 0} \dfrac{\dfrac{1}{5 + h} - \dfrac{1}{5}}{h}$

(c) $\lim\limits_{x \to 64} \dfrac{\sqrt{x} - 8}{x - 64}$

(d) $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{\cos^2 x - 3 \cos x + 2}$

3. 10 pts. Suppose
$$G(x) = \begin{cases} 3x - 4k, & x < -3 \\ x + 9, & x > -3. \end{cases}$$

Determine a value for k for which the limit $\lim\limits_{x \to -3} G(x)$ exists, and state the value of the limit.

4. 4 pts. each Determine the following limits:

$$\lim\limits_{x \to 3^+} \dfrac{-5}{(x - 3)^3}, \qquad \lim\limits_{t \to -2^-} \dfrac{t^3 - 5t^2 + 6t}{t^4 - 4t^2}, \qquad \lim\limits_{\theta \to 0^-} \cot 6\theta.$$

5. [10 pts.] Find all vertical asymptotes $x = a$ of the function

$$f(x) = \frac{x+1}{x^3 - 4x^2 + 4x}.$$

For each value of a determine $\lim_{x \to a^+} f(x)$, $\lim_{x \to a^-} f(x)$, and $\lim_{x \to a} f(x)$.

6. [10 pts.] Evaluate the limit

$$\lim_{x \to -\infty} \frac{8 - 4x^2}{7x^2 + 5x - 12}.$$

7. [15 pts.] Determine $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ for

$$f(x) = \frac{4x^3 + 1}{2x^3 + \sqrt{16x^6 + 1}}.$$

Then give the horizontal asymptotes of f, if any.

8. [10 pts.] Let h be given by

$$h(x) = \begin{cases} 2x^2 - x, & \text{if } x < -1 \\ s, & \text{if } x = -1 \\ 3x - 5, & \text{if } x > -1 \end{cases}$$

Find the value of s for which h is continuous from the left at -1, and the value of s for which h is continuous from the right at -1.

9. Let $f(x) = \sqrt{2x - 1}$.

   (a) [10 pts.] Use the limit definition of a derivative to find $f'(1)$.

   (b) [5 pts.] Determine an equation for the tangent line to the graph of f at the point $(1, 1)$.

10. [15 pts.] Use the limit definition of a derivative to find $f'(x)$ given that

$$f(x) = \frac{1}{3x - 4}.$$