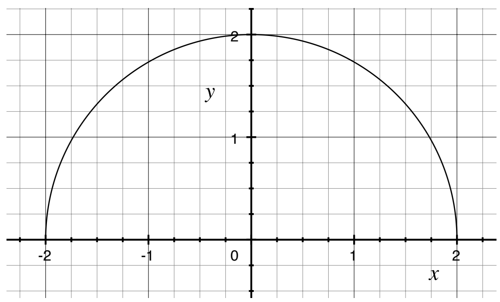
MATH 140 EXAM #4 KEY (SUMMER II - 2010)

1. Area $= \sum_{i=1}^{5} f(x_i)\cdot\Delta x = \sum_{i=1}^{5}(25-i^2) = \sum_{i=1}^{5} 25 - \sum_{i=1}^{5} i^2 = 25(5) - \frac{5(6)(11)}{6} = 70.$

2. $\Delta x = \frac{4}{n}$ and $x_i = -1 + i(\frac{4}{n}) = \frac{4i}{n} - 1$. So, with $f(x) = 1+x^2$, we have: $\displaystyle\int_{-1}^{3}(1+x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left(1+x_i^2\right)\cdot\frac{4}{n} =$ $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left[1+\left(\frac{4i}{n}-1\right)^2\right]\cdot\frac{4}{n} = \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left[2-\frac{8}{n}i+\frac{16}{n^2}i^2\right] = \lim_{n\to\infty}\frac{4}{n}\left[2n-\frac{8}{n}\cdot\frac{n(n+1)}{2}+\frac{16}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right] =$ $\displaystyle\lim_{n\to\infty}\frac{4}{n}\left(\frac{10n^2+12n+8}{3n}\right) = \lim_{n\to\infty}\left(\frac{40n^2+48n+32}{3n^2}\right) = \frac{40}{3}$

3. The graph of $y=\sqrt{4-x^2}$ is below, and is seen to be the upper half of a circle with radius 2. So $\int_{-2}^{2}\sqrt{4-x^2}\,dx = \frac{1}{2}\cdot\pi(2)^2 = 2\pi$



4a. $F'(x) = \frac{d}{dx}[-\int_{10}^{x}\sin^4 t\,dt] = \sin^4 x.$

4b. Let $\phi(x) = \int_1^x (t+\tan t)dt$, so $\phi'(x) = x+\tan x$, so $y=\phi(\cos x)$ by the first part of the Fundamental Theorem of Calculus, and by the Chain Rule we get $y' = \phi'(\cos x)\cdot\frac{d}{dx}(\cos x) = (\cos x + \tan(\cos x))\cdot(-\sin x)$, or equivalently $y' = -\sin x(\cos x + \tan(\cos x))$.

5a. $= [-4\cos\theta - 3\sin\theta]_0^\pi = [-4\cos\pi - 3\sin\pi] - [-4\cos 0 - 3\sin 0] = 8.$

5b. $\int_{-4}^{0}(3s-2|s|)ds + \int_0^2(3s-2|s|)ds = \int_{-4}^{0}(3s-2(-s))ds + \int_0^2(3s-2s)ds = \int_{-4}^{0}5s\,ds + \int_0^2 s\,ds = \frac{5}{2}s^2|_{-4}^{0} + \frac{1}{2}s^2|_0^2 = \frac{5}{2}(0-(-4)^2) + \frac{1}{2}(2^2-0) = -38.$

5c. Let $u = x-1$, so that $x = u+1$ and $du = dx$. When $x=1$ we have $u=0$, and when $x=2$ we have $u=1$. With this substitution the integral becomes $\int_0^1(u+1)\sqrt{u}\,du = \int_0^1(u^{3/2}+u^{1/2})du = [\frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}]_0^1 = \frac{2}{5}+\frac{2}{3} = \frac{16}{15}.$

5d. Let $u=\phi^2$. So $\frac{du}{dx} = 2\phi \;\Rightarrow\; \phi d\phi = \frac{1}{2}du$. When $\phi = 0$ we have $u=0$ also, and when $\phi=\sqrt{\pi}$ we have $u=\pi$. Now integral becomes $\int_0^\pi \frac{1}{2}\cos u\,du = \frac{1}{2}[\sin u]_0^\pi = \frac{1}{2}(\sin\pi - \sin 0) = 0.$

6a. Let $u=\sin\theta$, so $\frac{du}{dx} = \cos\theta \;\Rightarrow\; du = \cos\theta dx$. Integral becomes $\int u^6\,du = \frac{1}{7}u^7 + C = \frac{1}{7}\sin^7\theta + C.$

6b. Let $u = 1+x+2x^2$, so $\frac{du}{dx} = 1+4x$ by some extraordinary coincidence, and then $(1+4x)dx = du$ so that our integral becomes $\int\frac{1}{\sqrt{u}}du = \int u^{-1/2}\,du = 2u^{1/2}+C = 2\sqrt{1+x+2x^2}+C.$

6c. Let $u = \cot x$. So $\frac{du}{dx} = -\csc^2 x$, which gives $\csc^2 x\, dx = -du$. Now, integral becomes $\int -\sqrt{u}\, du = -\frac{2}{3}u^{3/2} + C = -\frac{2}{3}(\cot x)^{3/2} + C = -\frac{2}{3}\sqrt{\cot^3 x} + C$.

7. First we need to find where the two curves $f(x) = \sqrt{x+3}$ and $g(x) = \dfrac{x+3}{2}$ intersect. Set $\sqrt{x+3} = \dfrac{x+3}{2}$ to get $x^2 + 2x - 3 = 0$ and finally $x = -3, 1$. For $-3 \leqslant x \leqslant 1$ we find that $f(x) \geqslant g(x)$, so the area is found to be $\displaystyle\int_{-3}^{1} [f(x) - g(x)]dx = \int_{-3}^{1} \left(\sqrt{x+3} - \frac{x+3}{2}\right) dx = \left[\frac{2}{3}(x+3)^{3/2} - \frac{1}{4}x^2 - \frac{3}{2}x\right]_{-3}^{1} = \frac{43}{12} - \frac{9}{4} = \frac{4}{3}$.

8. $\displaystyle V = \int_0^1 \left[\pi x^2 - \pi(x^3)^2\right] dx = \pi \int_0^1 (x^2 - x^6)dx = \pi \left[\frac{1}{3}x^3 - \frac{1}{7}x^7\right]_0^1 = \pi\left(\frac{1}{3} - \frac{1}{7}\right) = \frac{4\pi}{21}$.