MATH 140 EXAM #2 KEY (SUMMER II - 2010)

1. $g'(x) = \lim_{h\to 0} \dfrac{\sqrt{2-3(x+h)} - \sqrt{2-3x}}{h} = \lim_{h\to 0} \dfrac{\sqrt{2-3(x+h)} - \sqrt{2-3x}}{h} \cdot \dfrac{\sqrt{2-3(x+h)} + \sqrt{2-3x}}{\sqrt{2-3(x+h)} + \sqrt{2-3x}}$

$= \lim_{h\to 0} \dfrac{[2-3(x+h)] - (2-3x)}{h[\sqrt{2-3(x+h)} + \sqrt{2-3x}\,]} = \lim_{h\to 0} \dfrac{-3h}{h[\sqrt{2-3(x+h)} + \sqrt{2-3x}\,]} = \lim_{h\to 0} \dfrac{-3}{\sqrt{2-3x-3h} + \sqrt{2-3x}}$

$\overset{\text{DST}}{=\!=} \dfrac{-3}{\sqrt{2-3x} + \sqrt{2-3x}} = \boxed{-\dfrac{3}{2\sqrt{2-3x}}}$. So Dom $g = \left(-\infty, \frac{2}{3}\right]$, and Dom $g' = \left(-\infty, \frac{2}{3}\right)$.

2a. We have $u = t^{1/5} + 4t^{5/2}$, so $u' = \boxed{\frac{1}{5}t^{-4/5} + 10t^{3/2}}$

2b. $y' = \dfrac{(t^4+1)(2t) - (t^2+2)(4t^3)}{(t^4+1)^2} = \boxed{\dfrac{2t - 8t^3 - 2t^5}{(t^4+1)^2}}$

2c. $f'(x) = \dfrac{(2-\tan x)(1) - (x)(0 - \sec^2 x)}{(2-\tan x)^2} = \boxed{\dfrac{2 - \tan x + x\sec^2 x}{(2-\tan x)^2}}$

2d. $g'(x) = 100(4x - x^2)^{99} \cdot \dfrac{d}{dx}(4x - x^2) = \boxed{100(4x-x^2)^{99} \cdot (4-2x) = 200(2-x)(4x-x^2)^{99}}$

2e. $y' = \cos(x\cos x) \cdot \dfrac{d}{dx}(x\cos x) = \boxed{\cos(x\cos x) \cdot (\cos x - x\sin x)}$

2f. $y' = \frac{1}{2}\left(x + \sqrt{x}\,\right)^{-1/2} \cdot \dfrac{d}{dx}\left(x + \sqrt{x}\,\right) = \frac{1}{2}\left(x + \sqrt{x}\,\right)^{-1/2} \cdot \left(1 + \frac{1}{2}x^{-1/2}\right) = \dfrac{1}{2\sqrt{x+\sqrt{x}}} \cdot \dfrac{2\sqrt{x}+1}{2\sqrt{x}} = \boxed{\dfrac{2\sqrt{x}+1}{4\sqrt{x^2 + x\sqrt{x}}}}$

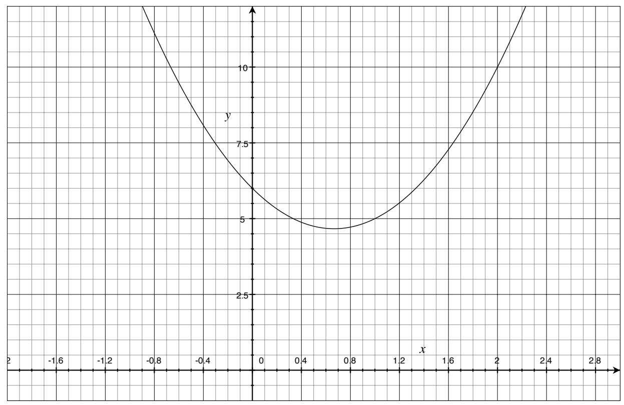
3. $H'(t) = \boxed{3\sec^2 3t}$ and $H''(t) = 6\sec 3t \cdot \dfrac{d}{dt}(\sec 3t) = 6\sec 3t \cdot \sec 3t \tan 3t \cdot 3 = \boxed{18\sec^2 3t \tan 3t}$.

4. The slope of the tangent line will be $y'(1)$ and the slope of the normal line will be $-1/y'(1)$. From $y'(x) = 4(1+2x)$ we get $y'(1) = 12$. Using the point-slope formula with $(x_1, y_1) = (1, 9)$ and $m = 12$ gives $y - 9 = 12(x-1)$ for the equation of the tangent line, or $\boxed{y = 12x - 3}$. The equation of the normal line is $y - 9 = -\frac{1}{12}(x-1)$, or $\boxed{y = -\frac{1}{12}x + \frac{109}{12}}$.

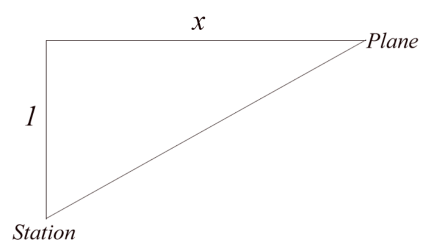
5. Implicit differentiation of the equation yields $(x^2 \cdot 2yy' + y^2 \cdot 2x) + (x \cdot \cos y \cdot y' + \sin y) = 0$, which yields $(2x^2y + x\cos y)y' = -2xy^2 - \sin y$, and finally $y' = \boxed{-\dfrac{2xy^2 + \sin y}{2x^2y + x\cos y}}$.

6. Differentiating both sides of the equation gives $2x + 2xy' + 2y - 2yy' + 1 = 0$, which leads with a little algebra to the equation $(2y-2x)y' = 2x+2y+1$, and thus $y'(x,y) = \dfrac{2x+2y+1}{2y-2x}$. The symbol $y'(x,y)$ means y', but it emphasizes that y' is actually a function of the variables x and y. Now, the slope of the tangent line is $y'(1,2) = \dfrac{2(1)+2(2)+1}{2(2)-2(1)} = \dfrac{7}{2}$. By the point-slope formula we obtain $y - 2 = \dfrac{7}{2}(x-1)$ for the equation of the tangent line, or $\boxed{y = \dfrac{7}{2}x - \dfrac{3}{2}}$.

7. The function $Q'(t) = 3t^2 - 4t + 6$ gives the *rate* at which the quantity of charge passing through the specified point in the wire is *changing*, which is called the current. The current at 0.5 second is $Q'(0.5) = 3(0.5)^2 - 4(0.5) + 6 = \boxed{4.75 \text{ A}}$, and at 1 second is $Q'(1) = \boxed{5.00 \text{ A}}$. To find when the current is lowest means to find the value of t that minimizes the value of $Q'(t)$. The curve $y = Q'(t)$ graphs as a parabola, so it all comes down to determining the lowest point on the parabola (i.e. the vertex) pictured below. This can (and really should) be done analytically. Applying the usual completing the square procedure gives $Q'(t) = 3\left(t - \frac{2}{3}\right)^2 + \frac{58}{9}$, which should make it clear that the minimum value of $Q'(t)$ is $Q'(2/3) = 58/9$. That is, the current is lowest at time $\boxed{t = 2/3 \text{ s}}$.



8. We find the distance between the plane and the radar station as a function of x, the horizontal distance depicted in the figure below. Ye Olde Pythagorean Theorem informs us that $D(x) = \sqrt{x^2 + 1}$. Now we find the rate of change of the distance between the plane and the station with respect to time t: $\frac{dD}{dt} = \frac{x}{\sqrt{x^2+1}} \cdot \frac{dx}{dt}$. We're given that $\frac{dx}{dt} = 500$ mi/h, so we have: $\left.\frac{dD}{dt}\right|_{x=2} = \frac{2}{\sqrt{2^2+1}} \cdot 500 = \boxed{447.2 \text{ mi/h}}$.



9. The diameter of the pile's base equals the height, which is to say the radius r equals $\frac{1}{2}h$ so that $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$. Differentiating with respect to t gives $\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt}$. But we're given that $\frac{dV}{dt} = 40 \text{ ft}^3/\text{min}$, so we have $\frac{dh}{dt} = \frac{160}{\pi h^2}$. Finally we can find the rate at which the height of the pile is changing over time when $h = 12$ ft: $\left.\frac{dh}{dt}\right|_{h=12} = \frac{160}{\pi 12^2} = \frac{10}{9\pi} = \boxed{0.354 \text{ ft/min}}$.