MATH 140 EXAM #2 KEY (SUMMER 2018)

1a $s'(x) = 48x^{11} - 2x^3$.

1b Since $f(x) = x^{1/2}(x^2 - 1) = x^{5/2} - x^{1/2}$, we have $f'(x) = \frac{5}{2}x^{3/2} - \frac{1}{2}x^{-1/2}$.

1c $g'(r) = \dfrac{(1+\sqrt{r})(2r) - r^2/(2\sqrt{r})}{(1+\sqrt{r})^2} = \dfrac{2r + 2r^{3/2} - \frac{1}{2}r^{3/2}}{(1+\sqrt{r})^2} = \dfrac{4r + 3r^{3/2}}{2(1+r^{1/2})^2}$.

1d $h'(\theta) = \theta(-\csc\theta\cot\theta) + \csc\theta - (-\csc^2\theta) = -\theta\csc\theta\cot\theta + \csc\theta + \csc^2\theta$.

1e $y' = \dfrac{1}{4}(1 + 2x + x^3)^{-3/4} \cdot (2 + 3x^2) = \dfrac{2 + 3x^2}{4(1 + 2x + x^3)^{3/4}}$.

1f $y' = \cos(\tan 2x) \cdot \sec^2(2x) \cdot 2 = 2\sec^2(2x)\cos(\tan 2x)$.

2 With the Quotient Rule,

$$f'(x) = -\frac{5}{(3x-2)^2} \quad \Rightarrow \quad f''(x) = \frac{30}{(3x-2)^3}.$$

3 Find all x for which $f'(x) = 60$, which is to say $6x^2 - 6x - 12 = 60$, and hence $x^2 - x - 12 = 0$. Solutions are $x = -3$ and $x = 4$, which corresponds to the points $(-3, -41)$ and $(4, 36)$.

4 Implicit differentiation yields

$$\frac{1}{2}(x + y^2)^{-1/2}(1 + 2yy') = y'\cos y,$$

where of course y' is dy/dx. The rest is boring algebra:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x + y^2}\cos y - 2y}.$$

5 Implicit differentiation yields

$$\frac{dy}{dx} = \frac{5}{2y - \cos y}.$$

The slope of the tangent line is therefore

$$m = \frac{5}{2\pi - \cos\pi} = \frac{5}{2\pi + 1}.$$

Equation of the tangent line is thus

$$y = \frac{5}{2\pi + 1}\left(x - \frac{\pi^2}{5}\right) + \pi.$$

6 Story problem! At time t the westbound car has gone a distance of $42t$ miles while the southbound car has gone a distance of $70t$ miles. These distances are the lengths of the two legs of a right triangle, and the distance between the cars, $d(t)$ equals the length of the hypotenuse. By the Pythagorean Theorem we have

$$d(t) = \sqrt{(42t)^2 + (70t)^2},$$

so the rate of change of the distance between the cars at time t is given by

$$d'(t) = \frac{1}{2}[(42t)^2 + (70t)^2]^{-1/2} \cdot [2 \cdot 42^2 t + 2 \cdot 70^2 t] = \frac{42^2 t + 70^2 t}{\sqrt{(42t)^2 + (70t)^2}}.$$

So at 3 hours the cars are moving apart at a rate of

$$d'(3) = \frac{42^2(3) + 70^2(3)}{\sqrt{(42 \cdot 3)^2 + (70 \cdot 3)^2}}.$$

This is good enough for an exam that does not allow a calculator, though it works out to about 81.6 mi/hr.

7 The diameter of the pile's base equals the height, which is to say the radius r equals $\frac{1}{2}h$ so that

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3.$$

Differentiating with respect to t gives

$$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt} \quad \Rightarrow \quad \frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt}.$$

But we're given that $dV/dt = 30$ ft^3/min, so we have $dh/dt = 120/\pi h^2$. Finally we can find the rate at which the height of the pile is changing over time when $h = 12$ ft:

$$\left.\frac{dh}{dt}\right|_{h=12} = \frac{120}{\pi 12^2} = \frac{5}{6\pi}$$