Math 140 Exam #2 Key (Summer 2013)

1a By the Product Rule,

$$f'(x) = (20x^3 + 6x)(x^3 + 7) + (5x^4 + 3x^2 + 1)(3x^2).$$

1b By the Quotient Rule,

$$g'(t) = \frac{(t^2 + 1)(2t) - (t^2 - 1)(2t)}{(t^2 + 1)^2} = \frac{4t}{(t^2 + 1)^2}.$$

1c Product Rule again:

$$y' = (\sin x)(-\sin x) + (\cos x)(\cos x) = \cos^2 x - \sin^2 x.$$

1d Quotient Rule again:

$$y' = \frac{(1 + \sin x)(2\cos x)' - (2\cos x)(1 + \sin x)'}{(1 + \sin x)^2} = \frac{(1 + \sin x)(-2\sin x) - (2\cos x)(\cos x)}{(1 + \sin x)^2},$$

or simply

$$y' = -\frac{2}{\sin x + 1}.$$

2a Velocity function: $v(t) = s'(t) = 6t^2 - 42t$. Setting $v(t) = 0$ gives

$$6t^2 - 42t = 0 \quad \Rightarrow \quad 6t(t - 7) = 0 \quad \Rightarrow \quad t = 0, 7.$$

However $t = 7$ is outside the designated domain $[0, 6]$, so the object is at rest only at time $t = 0$.

2b Acceleration function: $a(t) = v'(t) = s''(t) = 12t - 42$. Setting $a(t) = 0$ gives $t = 7/2$, the time when the acceleration is zero. Also $a(t) < 0$ for $t \in [0, 7/2)$ and $a(t) > 0$ for $t \in (7/2, 6]$.

3a By the Chain Rule,

$$f'(x) = \frac{1}{2}(x^3 + 4)^{-1/2}(3x^2) = \frac{3x^2}{2\sqrt{x^3 + 4}}.$$

3b Again by the Chain Rule,

$$g'(t) = \cos(9\cos t) \cdot (-9\sin t) = -9\cos(9\cos t)\sin t.$$

3c Applying the Chain Rule twice,

$$h'(z) = \sec^2(\sqrt{\sec z}) \cdot \frac{1}{2}(\sec z)^{-1/2} \cdot \sec z \tan z = \frac{\sec^2(\sqrt{\sec z}) \sec z \tan z}{2\sqrt{\sec z}}$$

4 Differentiating both sides,

$$3(xy+1)^2(xy'+y) = 1-2yy' \;\;\Rightarrow\;\; 3x(xy+1)^2y' + 2yy' = 1-3y(xy+1)^2$$
$$\Rightarrow\;\; y' = \frac{1-3y(xy+1)^2}{3x(xy+1)^2+2y}.$$

5 With implicit differentiation we have

$$9x^2 + 21y^2y' = 10y' \;\;\Rightarrow\;\; y'(x,y) = \frac{9x^2}{10-21y^2}.$$

The slope of the curve at $(1,1)$ is thus

$$y'(1,1) = \frac{9(1)^2}{10-21(1)^2} = -\frac{9}{11}.$$

Now, the equation of the tangent line is $y-1 = -\frac{9}{11}(x-1)$, or

$$y = -\frac{9}{11}x + \frac{20}{11}.$$

6 Area of rectangle at time t is

$$A(t) = (2+t)(4+t) = t^2+6t+8.$$

Rate of change of the area at time t is $A'(t) = 2t+6$. Thus at time $t=20$ seconds the area is increasing at a rate of $A'(20) = 2(20)+6 = 46$ cm^2/s.

7 The ground, wall, and ladder form a right triangle with hypotenuse of length 13. If x is the distance between the wall and the foot of the ladder, and y is the distance between the ground and the top of the ladder, then $x^2+y^2 = 13^2$. Both x and y are functions of time t, and so differentiating with respect to t yields

$$2x(t)x'(t) + 2y(t)y'(t) = 0 \;\;\Rightarrow\;\; y'(t) = -\frac{x(t)x'(t)}{y(t)}.$$

We're given that $x'(t) = 0.5$ ft/s for all $t \geq 0$, and of course we also have $y(t) = \sqrt{169-x^2(t)}$. Thus

$$y'(t) = -\frac{x(t)}{2\sqrt{169-x^2(t)}}.$$

Now, at the time t when $x(t) = 5$ ft, we obtain

$$y'(t) = -\frac{5}{2\sqrt{169-5^2}} = -\frac{5}{24};$$

that is, at the time t when the foot of the ladder is 5 ft from the wall, the top of the ladder is sliding down the wall at a rate of $-5/24$ ft/s.