

1a. We know $\sqrt[5]{32} = 2$, so get the equation of the tangent line to the curve f at $(32, 2)$. First, $f'(x) = \frac{1}{5}x^{-4/5} \Rightarrow f'(32) = \frac{1}{5}(32)^{-4/5} = \frac{1}{5} \cdot \frac{1}{16} = \frac{1}{80}$, so the slope of the line is $\frac{1}{80}$. Now, using point-slope formula, we get $y - 2 = \frac{1}{80}(x - 32) \Rightarrow L(x) = \frac{1}{80}x + \frac{8}{5}$.

1b. $\sqrt[5]{33} = f(33) \approx L(33) = \frac{1}{80}(33) + \frac{8}{5} = 2.0125$. (Actual value is 2.012346617..., which amounts to only a 0.0076% error.)

2. Let $f(x) = 2x - 1 - \sin x$, so $f(0) = -1 < 0$ and $f(\pi) = 2\pi - 1 > 0$. Since f is continuous on $[0, \pi]$ and differentiable on $(0, \pi)$, the Intermediate Value Theorem implies there exists some $r \in (0, \pi)$ such that $f(r) = 0$. Now suppose that f has two real zeros r_1 and r_2 , with $r_1 < r_2$. Since $f(r_1) = 0 = f(r_2)$, f is continuous on $[r_1, r_2]$, and f is differentiable on (r_1, r_2) , by Rolle's Theorem there exists some $c \in (r_1, r_2)$ such that $f'(c) = 0$. Since $f'(x) = 2 - \cos x$, we obtain $\cos c = 2$. But this is a contradiction, since the cosine function cannot equal 2 at any real number! Hence f cannot have *more than* one real zero, and since it's been shown to have *at least* one real zero, we conclude that it must have *exactly* one real zero. Therefore $2x - 1 - \sin x = 0$ has exactly one real root.

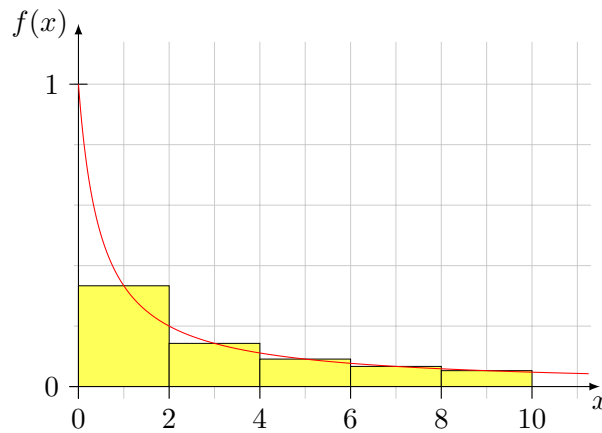
3a. $\int (3x^{-2} - 4x^2 + 1)dx = -3x^{-1} - \frac{4}{3}x^3 + x + C$

3b. $\int [\cos(4t) - \sin(t/4)]dt = \frac{1}{4}\sin(4t) + 4\cos(t/4) + C$

3c. Let $u = 1 - 4x^3$, so by the mechanism of “ u -substitution” we get $du = -12x^2dx \Rightarrow -\frac{1}{6}du = 2x^2dx$, and so $\int \frac{2x^2}{\sqrt{1-4x^3}}dx = -\frac{1}{6} \int \frac{1}{\sqrt{u}}du = -\frac{1}{6} \int u^{-1/2}du = -\frac{1}{6} \cdot 2u^{1/2} = -\frac{1}{3}\sqrt{1-4x^3} + C$.

4. $f(x) = \int f'(x)dx = \int (8x - 5)dx = 4x^2 - 5x + C$, which implies that $f(0) = C$. But we're given $f(0) = 4$, so we obtain $C = 4$ and arrive at the solution $f(x) = 4x^2 - 5x + 4$.

5. Area $\approx 2[f(1) + f(3) + f(5) + f(7) + f(9)] = 2(1/3 + 1/7 + 1/11 + 1/15 + 1/19) = 10,042/7,315 \approx 1.3728$ m.



6. $\lim_{\Delta \rightarrow 0} \sum_{k=1}^n \bar{x}_k \cos \bar{x}_k \Delta x_k = \int_1^2 x \cos x dx$

$$7a. \int_0^3 5f(x) dx = 5 \int_0^3 f(x) dx = 5(2) = 10$$

$$7b. \int_3^6 [3f(x) - g(x)] dx = 3 \int_3^6 f(x) dx - \int_3^6 g(x) dx = 3(-9) - 5 = -32$$

$$7c. \int_6^3 [f(x) + 2g(x)] dx = - \int_3^6 [f(x) + 2g(x)] dx = -[(-9) + 2(5)] = -1$$

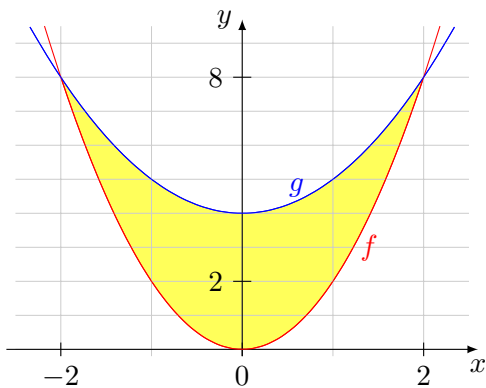
8. We can partition $[3, 7]$ into n subintervals each of length $\Delta x = \frac{7-3}{n} = \frac{4}{n}$, and evaluate $f(x) = 4x + 6$ at the right endpoint of each subinterval, so $x_i^* = 3 + \frac{4}{n}i$. By definition, $\int_3^7 f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x_i = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[4 \left(3 + \frac{4}{n}i \right) + 6 \right] \cdot \frac{4}{n} = \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left(18 + \frac{16}{n}i \right) = \lim_{n \rightarrow \infty} \frac{4}{n} \left[18n + \frac{16}{n} \cdot \frac{n(n+1)}{2} \right] = \lim_{n \rightarrow \infty} \frac{4}{n} (26n + 8) = \lim_{n \rightarrow \infty} \left(104 + \frac{32}{n} \right) = 104.$

$$9. \frac{d}{dx} \int_9^{x^3} \sin^5(t) dt = \sin^5(x^3) \cdot 3x^2 = 3x^2 \sin^5(x^3).$$

$$10a. \int_1^4 \frac{5t^6 - \sqrt{t}}{t^2} dt = \int_1^4 (5t^4 - t^{-3/2}) dt = \left[t^5 + 2t^{-1/2} \right]_1^4 = (1024 + 1) - (1 + 2) = 1022.$$

10b. Let $u = \sin \theta$, so $du = \cos \theta d\theta$. When $x = 0$ we get $u = 0$ also; and when $x = \pi/2$ we get $u = 1$. Thus we obtain $\int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta = \int_0^1 u^2 du = \left[\frac{1}{3} u^3 \right]_0^1 = \frac{1}{3}.$

11. First we find the points where the curves generated by f and g intersect: $f(x) = g(x) \Rightarrow 2x^2 = x^2 + 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$, so the points are $(-2, f(-2)) = (-2, 8)$ and $(2, f(2)) = (2, 8)$. Between $x = -2$ and $x = 2$ we have $f(0) = 0 < 4 = g(0)$, so $g(x) \geq f(x)$ when $-2 \leq x \leq 2$. We now find the area of the bounded region: $\text{Area} = \int_{-2}^2 [g(x) - f(x)] dx = \int_{-2}^2 (4 - x^2) dx = \left[4x - \frac{1}{3} x^3 \right]_{-2}^2 = (8 - 8/3) - (-8 + 8/3) = 16 - 16/3 = \frac{32}{3}.$



12. In the first quadrant the curve given by $f(x) = 4 - x^2$ goes from $(0, 4)$ to $(2, 0)$, so the limits of integration will be $x = 0$ and $x = 2$: $\text{Volume} = \int_0^2 \pi [f(x)]^2 dx = \pi \int_0^2 (4 - x^2)^2 dx = \pi \int_0^2 (16 - 8x^2 + x^4) dx = \pi \left[16x - \frac{8}{3} x^3 + \frac{1}{5} x^5 \right]_0^2 = \pi \left[16(2) - \frac{8}{3}(8) + \frac{1}{5}(32) \right] = \frac{256}{15} \pi.$