**MATH 140 EXAM #4 KEY (SUMMER 2010)**

**1.** $\int_2^{10} \sqrt{x^3+1}\,dx \;\approx\; \Delta x\,[\,f(3)+f(5)+f(7)+f(9)\,] \;=\; \dfrac{10-2}{4}\cdot\left[\sqrt{3^3+1}+\sqrt{5^3+1}+\sqrt{7^3+1}+\sqrt{9^3+1}\right] =$ $2\left(\sqrt{28}+\sqrt{126}+\sqrt{344}+\sqrt{730}\right)\approx 124.1644.$

**2.** $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$ and $x_i = 0+i\left(\dfrac{2}{n}\right) = \dfrac{2i}{n}$, so by definition $\int_0^2 (2-x^2)dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[2-\left(\dfrac{2i}{n}\right)^2\right]\left(\dfrac{2}{n}\right) =$ $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\dfrac{4}{n}-\dfrac{8}{n^3}i^2\right) = \lim\left(\sum\dfrac{4}{n}-\dfrac{8}{n^3}\sum i^2\right) = \lim\left(4-\dfrac{8}{n^3}\cdot\dfrac{n(n+1)(2n+1)}{6}\right) = \lim\left(\dfrac{4n^2-12n-4}{3n^2}\right) = \dfrac{4}{3}.$

**3a.** $h'(r) = \dfrac{d}{dr}\int_9^r t^2\sin t\,dt = r^2\sin r.$

**3b.** $y' = \sqrt{\tan x+\sqrt{\tan x}}\cdot\dfrac{d}{dx}(\tan x) = \sec^2 x\sqrt{\tan x+\sqrt{\tan x}}.$

**4a.** $= \int_4^{16}\left(x^{1/2}-x^{-1/2}\right)dx = \left[\dfrac{2}{3}x^{3/2}-2x^{1/2}\right]_4^{16} = \left(\dfrac{2}{3}(16)^{3/2}-2(16)^{1/2}\right)-\left(\dfrac{2}{3}(4)^{3/2}-2(4)^{1/2}\right) = \dfrac{100}{3}.$

**4b.** $= \int_0^{\pi/4}(\sec^2\theta+1)d\theta = [\tan\theta+\theta]_0^{\pi/4} = \left(\tan\dfrac{\pi}{4}+\dfrac{\pi}{4}\right)-0 = 1+\dfrac{\pi}{4}.$

**4c.** $= \int_{-3}^{0}[x-2(-x)]dx+\int_0^2[x-2(x)]dx = \int_{-3}^{0}3x\,dx-\int_0^2 x\,dx = \left[\tfrac{3}{2}x^2\right]_{-3}^{0}-\left[\tfrac{1}{2}x^2\right]_0^2 = \left[0-\tfrac{3}{2}(-3)^2\right]-\left[\tfrac{1}{2}(2)^2-0\right] =$ $-\tfrac{31}{2}.$

**4d.** Let $u=1+2x^3$, so $du=6x^2dx$ and we get $x^2dx=\tfrac{1}{6}du$. When $x=0$ we have $u=1$, and when $x=1$ we have $u=3$. Then our integral becomes: $\int_1^3 u^5\cdot\tfrac{1}{6}du = \tfrac{1}{6}\int_1^3 u^5du = \tfrac{1}{6}\left[\tfrac{1}{6}u^6\right]_1^3 = \tfrac{1}{36}(3^6-1^6) = \tfrac{182}{9}.$

**4e.** The integral equals 0 since $\tan^3\varphi$ is an odd function and the limits of integration are opposite values.

**5a.** Let $u=\sin\theta$, so $du=\cos\theta d\theta$ and we get: $\int\cos\theta\sin^6\theta\,d\theta = \int u^6\,du = \tfrac{1}{7}u^7+C = \tfrac{1}{7}\sin^7\theta+C.$

**5b.** Let $u=2x+x^2$, so $du=(2+2x)dx$ gives $(x+1)dx=\tfrac{1}{2}du$, and we obtain $\int(x+1)\sqrt{2x+x^2}\,dx = \int\sqrt{u}\cdot\tfrac{1}{2}du =$ $\tfrac{1}{2}\int u^{1/2}du = \tfrac{1}{2}(\tfrac{2}{3}u^{3/2})+C = \tfrac{1}{3}(2x+x^2)^{3/2}+C.$

**6.** Let $f(x)=\sqrt{x+3}$ and $g(x)=\dfrac{x+3}{2}$. Setting $f(x)=g(x)$ yields $\sqrt{x+3}=\dfrac{x+3}{2}$, which has solutions $x=-3,1$. So the curves $f$ and $g$ intersect at the points $(-3,0)$ and $(1,2)$, and on the interval $[-3,1]$ we have $f(x)\geq g(x)$. Area $=\int_{-3}^{1}[f(x)-g(x)]dx = \int_{-3}^{1}(\sqrt{x+3}-\tfrac{x+3}{2})dx = \left[\tfrac{2}{3}(x+3)^{3/2}-\tfrac{1}{4}x^2-\tfrac{3}{2}x\right]_{-3}^{1} = \tfrac{43}{12}-\tfrac{9}{4} = \tfrac{4}{3}.$

**7.** $V = \int_0^1\left[\pi x^2-\pi(x^3)^2\right]dx = \pi\int_0^1(x^2-x^6)dx = \pi\left[\tfrac{1}{3}x^3-\tfrac{1}{7}x^7\right]_0^1 = \pi(\tfrac{1}{3}-\tfrac{1}{7}) = \tfrac{4\pi}{21}.$