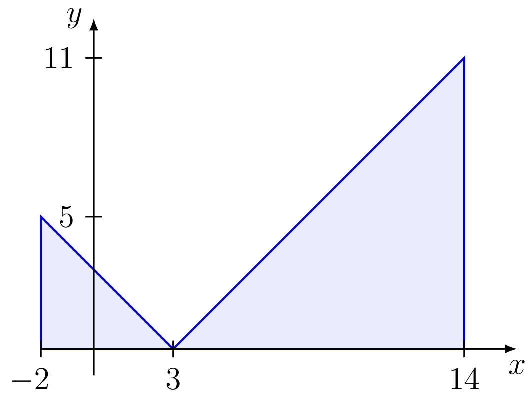
Math 140 Exam #4 Key (Spring 2024)

1a Find the areas of the triangles below: integral equals 73.



1b The area under the curve $y = f(x)$ is a 4-by-5 rectangle and a triangle with base 2 and height 6. Integral equals 26.

2a Expanding the product, we have

$$\int_0^4 (t^3 - 6t^2 + 8t)\,dt = \left[\frac{1}{4}t^4 - 2t^3 + 4t^2\right]_0^4 = 0.$$

2b $2\int_1^8 x^{1/3} dx = 2\left[\frac{3}{4}x^{4/3}\right]_1^8 = \frac{45}{2}.$

2c $\int_{\pi/4}^{3\pi/4} \csc^2\theta\,d\theta = \big[-\cot\theta\big]_{\pi/4}^{3\pi/4} = \cot\frac{\pi}{4} - \cot\frac{3\pi}{4} = 1 - (-1) = 2.$

3 By the Fundamental Theorem of Calculus and Chain Rule:

$$\frac{d}{dx}\int_{\cos x}^{9} \frac{6}{\sqrt{t^6+9}}\,dt = -\frac{d}{dx}\int_9^{\cos x} \frac{6}{\sqrt{t^6+9}}\,dt = -\frac{6}{\sqrt{\cos^6 x + 9}} \cdot (\cos x)' = \frac{6\sin x}{\sqrt{\cos^6 x + 9}}.$$

4a Let $u = x^2 + 1$, giving $du/dx = 2x$ and thus $du = 2x\,dx$. The integral becomes

$$\int_1^5 \frac{1}{u^2}\,du = \left[-\frac{1}{u}\right]_1^5 = \frac{4}{5}.$$

4b Let $u = 10x + 7$, so $du = 10\,dx$ and the integral becomes

$$\int \frac{1}{10}\sec^2 u\,du = 2\sin u + C = \frac{1}{10}\tan u + C = \frac{1}{10}\tan(10x+7) + C.$$

4c Let $u = 3z + 2$, so $du = 3\,dz$ and $z = \frac{u-2}{3}$. The integral becomes

$$\int \left(\frac{u-2}{3} + 1\right)\sqrt{u}\cdot\frac{1}{3}du = \frac{1}{9}\int (u^{3/2} + u^{1/2})\,du = \frac{1}{9}\left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right] + C$$
$$= \frac{2}{45}(3z+2)^{5/2} + \frac{2}{27}(3z+2)^{3/2} + C.$$

5 The curves $y = x/4$ and $y = \sqrt[3]{x}$ intersect where $x/4 = \sqrt[3]{x}$, or equivalently $x^3 - 64x = 0$. Solutions are $x = -8, 0, 8$. Area A is

$$A = \int_{-8}^{0}\left(\frac{x}{4} - \sqrt[3]{x}\right)dx + \int_{0}^{8}\left(\sqrt[3]{x} - \frac{x}{4}\right)dx = 4 + 4 = 8.$$

6 Volume is

$$V = \int_0^3 A(x)\,dx = \int_0^3 \frac{1}{2}\pi\left(\frac{3-x}{2}\right)^2 dx = \frac{\pi}{8}\int_0^3 (x^2 - 6x + 9)\,dx = \frac{9\pi}{8}.$$

7 Using the disc method here, the volume is

$$V = \int_2^4 \pi\left(\sqrt{25 - x^2}\,\right)^2 dx = \frac{94}{3}\pi.$$